Not-for-Publication Appendix: On Fiscal Multipliers: Estimates from a Medium Scale DSGE Model

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January 23, 2013

1 Complete Set of Symmetric Equilibrium Conditions

$$x_t^c = c_t - b^c s_{t-1}^C \tag{A-1}$$

$$x_t^g = g_t - b^g s_{t-1}^G \tag{A-2}$$

$$k_{t+1} = (1-\delta)k_t + i_t \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right] \tag{A-3}$$

$$d_t U_x(x_t^c, h_t) = \lambda_t \tag{A-4}$$

$$-d_t U_h(x_t^c, h_t) = \frac{\lambda_t w_t}{\tilde{\mu}_t}, \tag{A-5}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1} \left[(1-\tau_{t+1}^k) r_{t+1}^k u_{t+1} - a(u_{t+1}) + q_{t+1}(1-\delta) + \delta q_{t+1} \tau_{t+1}^k u_{t+1}\right] \tag{A-6}$$

$$\lambda_t = \lambda_t q_t \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right) \mathcal{S}'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta E_t \lambda_{t+1} q_{t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 \mathcal{S}'\left(\frac{i_{t+1}}{i_t}\right) \tag{A-7}$$

$$(1-\tau_t^k) r_t^k + \delta q_t \tau_t^k = a'(u_t) \tag{A-8}$$

$$(\tilde{\eta}-1)(1-\tau_t^w) h_t + \alpha^W \pi_t^w (\pi_t^w - \bar{\pi}) - \tilde{\eta} \frac{h_t}{\tilde{\mu}_t} = \alpha^W \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^w (\pi_{t+1}^w - \bar{\pi})\right] \tag{A-9}$$

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{A-10}$$

$$\frac{1 - mc_t - \tilde{\nu}_t^c}{\theta^c - 1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[b^c \tilde{\nu}_{t+1}^c + \frac{\theta^c}{\theta^c - 1} \left\{1 - mc_{t+1} - \tilde{\nu}_{t+1}^c\right\}\right] \tag{A-11}$$

$$\frac{1 - mc_t - \tilde{\nu}_t^g}{\theta^g - 1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[b^g \tilde{\nu}_{t+1}^g + \frac{\theta^g}{\theta^g - 1} \left\{1 - mc_{t+1} - \tilde{\nu}_{t+1}^g\right\}\right] \tag{A-12}$$

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$$1 - mc_t - \tilde{\nu}_t^i = 0 \tag{A-13}$$

$$\eta\left(\tilde{\nu}_t^c x_t^c + \tilde{\nu}_t^g x_t^g + \tilde{\nu}_t^i (y_t - c_t - g_t)\right) + \alpha^P \pi_t\left(\pi_t - \bar{\pi}\right) - y_t = \alpha^P \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\pi_{t+1}\left(\pi_{t+1} - \bar{\pi}\right)\right] \tag{A-14}$$

$$z_t F(u_t k_t, h_t) - \psi = c_t + g_t + i_t + a(u_t)k_t + \frac{\alpha^P}{2}(\pi_t - \pi)^2 + \frac{\alpha^W}{2}\left(\pi_t^w - \bar{\pi}\right)^2 w_t \tag{A-15}$$

$$y_t = z_t F(u_t k_t, h_t) - \psi - \frac{\alpha^P}{2}(\pi_t - \pi)^2 \tag{A-16}$$

$$\mathrm{mc}_t z_t F_2(u_t k_t, h_t) = w_t \tag{A-17}$$

$$\mathrm{mc}_t z_t F_1(u_t k_t, h_t) = r_t^k \tag{A-18}$$

$$b_t = R_{t-1}\frac{b_{t-1}}{\pi_t} + g_t + tr_t - \tau_t \tag{A-19}$$

$$\tau_t = \tau_t^w w_t h_t + \tau_t^k (r_t^k u_t k_t - \delta q_t u_t k_t) \tag{A-20}$$

$$s_t^C = \theta^c s_{t-1}^C + (1 - \theta^c)c_t \tag{A-21}$$

$$s_t^G = \theta^g s_{t-1}^G + (1 - \theta^g)g_t \tag{A-22}$$

and equations (2), (8), (13), (16), (17), (18), (19), and (21) from the text.

2 Steady State

$$q = 1,\ u = 1$$

$$R = \frac{\pi}{\beta}$$

$$r^k = (\frac{1}{\beta} - 1 + \delta - \delta\tau^k)/(1 - \tau^k)$$

$$share_i = I/Y = \delta\theta/r^k$$

$$share^c = C/Y = 1 - share^i - share^g$$

$$\gamma_1 = (1 - \tau^k)r^k + \delta\tau^k,\ \gamma_2 = \sigma^u\gamma_1$$

$$\tilde{\mu} = \frac{\tilde{\eta}}{(\tilde{\eta} - 1)(1 - \tau^w)}$$

$$mc = \frac{share^c + share^i + share^g}{\eta\left(share^c aa^c/bb^c + share^g aa^g/bb^g - share^i\right)} + 1$$

$$aa^c = (1 - b^c),\ bb^c = (\beta b^c(\theta^c - 1))/(\beta\theta^c - 1) - 1$$

$$aa^g = (1 - b^g),\ bb^g = (\beta b^g(\rho^g - 1))/(\beta\rho^g - 1) - 1$$

$$\nu^c = (mc - 1)/bbc,\ \nu^g = (mc - 1)/bbg,\ \nu^i = (mc - 1)$$

$$K = (r^k/mc/\theta)^{\frac{1}{\theta-1}} H$$

$$I = \delta K$$

$$w = mc(1-\theta)(K/H)^{\theta}$$

$$\psi = K^{\theta} H^{1-\theta} - (r^k K + wH)$$

$$Y = K^{\theta} H^{1-\theta} - \psi$$

$$s^c = C,\ s^g = G$$

$$a = (1-h)w/((1-H)w + \tilde{\mu}(C - b^c s^c))$$

$$\lambda = (1-a)(c(1-b^c))^{(1-a)(1-\gamma)-1}(1-h)^{a(1-\gamma)}$$

$$\tau = \tau^w wH + \tau^k (r^k - \delta)K$$

$$tr = b\left(1 - \frac{R}{\pi}\right) - G + \tau$$

3 Data used in estimation

The following quarterly series were used in the estimation. In order to construct real per-capita values, GDP deflator (given by Table 1.1.6, Line 1) and civilian non-institutional population, over 16 (given by LNU00000000Q, at Bureau of Labor Statistics) are used. The table and line numbers refer to the NIPA tables on the Bureau of Economic Analysis website. The data for consumption, investment, government spending and debt were linearly detrended to get stationary series.

- **Consumption**: Sum of personal consumption expenditures on non-durables goods (Table 1.1.5, Line 3) and services (Table 1.1.5, Line 5) divided by the GDP deflator and by population.

- **Inflation**: First difference of GDP deflator.

- **Federal funds rate**: Monthly federal funds rate series from St. Louis FRED website was averaged to create quarterly series.

- **Investment**: Sum of gross private domestic investment (Table 1.1.5, Line 6) and personal consumption expenditures on nondurable goods (Table 1.1.5, Line 4), divided by the GDP deflator and by population.

- **Government spending**: Government consumption expenditures and gross investment (Table 1.1.5, Line 20) divided by the GDP deflator and by population.

- **Debt**: Market value of federal debt held by public from the Dallas Fed website divided by the GDP deflator and by population. The quarterly series is constructed by summing up the monthly series. The series of debt initialized by the Dallas Fed series and constructed from secondary deficit data from NIPA matches up in levels and the correlation is 0.99.

- **Capital and labor tax rate**: The method of Jones (2002) was used to construct these series. The first step is to construct the average personal income tax rate,

$$\tau_p = \frac{FIT + SIT}{W + PRI/2 + CI}$$

where FIT denotes federal income taxes (Table 3.2, Line 3), SIT denotes state and local income taxes (Table 3.3, Line 3), W denotes wages and salaries (Table 1.12, Line 3), PRI denotes proprietor's income (Table 1.12, Line 9) and CI denotes capital income which is the sum of rental income (Table 1.12, Line 12), corporate profits (Table 1.12, Line 13), net interest (Table 1.12, Line 18) and $PRI/2$. The labor tax rate, τ^w, is then calculated as,

$$\tau^w = \frac{\tau^p[W + PRI/2] + CSI}{EC + PRI/2}$$

where CSI is total contributions to government social insurance (Table 3.1, Line 7) and EC denotes total compensation of employees (Table 1.12, Line 2). The capital tax rate, τ^k is calculated as,

$$\tau^k = \frac{\tau^p CI + CT + PT}{CT + PT}$$

The tax rates are constructed as average tax rates using the methodology in Mendoza, Razin and Tesar (1994) and Jones (2002), primarily because they are easily constructed on a quarterly basis using data on actual tax payments and national accounts, and in addition allow us to distinguish between taxes on labor and capital income. Other tax rate series include Barro and Sahasakul (1983) marginal tax rate series on personal income, where they average tax rates over the number of returns for each class of adjusted gross income. However, this does not differentiate between tax rates on capital and labor income. Mendoza, Razin and Tesar (1994) also show that average tax rates in different countries tend to follow the same dynamics as marginal tax rates.[1]

4 Calibration and Priors

Some of the parameters which are hard to identify or pin down in steady state are calibrated. These include the discount factor β, set at $1.03^{-1/4}$, which implies a steady-state annualized real interest rate of 3 percent. The depreciation rate, δ, is set at 0.025, which implies an annual rate of depreciation on capital equal to 10 percent. The production function parameter θ is set at 0.30, which corresponds to a steady state share of capital income roughly equal to 30 percent. The labor elasticity of substitution, $\tilde{\eta}$ is set at 21, and goods elasticity of substitution, η is set at 5.3, since with the introduction of deep habits the price markup movements are jointly determined by deep habit parameters and η is generally not well identified.

Steady state variables are also calibrated based on averages over the sample period considered in the paper. The share of government spending in aggregate output is set at 0.18, and the annual average of the ratio of debt to GDP pins down the steady state value to be 0.33. Similarly, the steady state values of the capital and labor tax rates are based on mean of the constructed series of average tax rates over the sample size, and are 0.41 and 0.23 respectively. Steady state labor is set at 0.5, which corresponds to a Frisch elasticity of labor supply of unity.

Table 1 in the paper shows the prior distribution for the parameters being estimated. These are consistent with the literature and the means of the distribution were set based on estimates from pre-existing studies. The autoregressive coefficients in the shock processes have a beta distribution

[1] Following Jones (2002), since the labor tax rate series has a trend and its idiosyncratic with no counterpart in the model, it is removed by linearly detrending the series. Mendoza, Razin and Tesar (1994) also show that in many different countries, the capital tax series is stationary but the labor tax series has an upward trend.

with a mean of 0.7 and standard deviation of 0.2. The only exception is the government spending process which is known to be highly persistent. The priors on standard deviations of the shocks have an inverse gamma distribution and are quite disperse. The deep habit parameters are assumed to have a beta distribution and the mean is in line with estimates from Zubairy (2009), where deep habits are explored as a transmission mechanism for government spending shocks with a limited information approach. The capacity utilization and investment adjustment cost parameters have normal distributions with means of 2.5 and 2 respectively, in line with estimates from Smets and Wouters (2007) and Altig et al. (2011). The coefficient of relative risk aversion γ is assumed to have a normal distribution with a mean of 2, which is higher than the logarithmic case. The nominal rigidity parameters have a normal distributions where the means correspond approximately with an adjustment frequency of close to four quarters, in the mapping between the Phillips curve coefficient implied by convex adjustment costs specification and the one with Calvo-Yun type rigidities. The standard deviation of these prior distributions are large to accommodate uncertainty in these parameters.

Monetary policy rule parameters have prior distributions similar to the ones adopted in Smets and Wouters (2007) and the mean values are also consistent with estimates from Clarida, Gali and Gertler (2000). On the other hand for fiscal policy rule parameters, the literature is less informative and so the priors are diffuse and span a larger parameter space. As mentioned above, the tax rate processes are assumed to be persistent. The tax rate elasticities to debt are assumed to have a gamma distribution with a mean of 0.5 and a standard deviation of 0.2, which is similar to Forni, Monteforte and Sessa (2009). The same priors are also used for the elasticities of spending and transfer response to the level of debt. In order to form priors on the response of the fiscal instruments to lagged output, I run a VAR(1) with tax rates and output. Based on this evidence the tax rate elasticities to output for both tax rates are thus assumed to have a gamma distribution with mean 0.15 and standard deviation of 0.1. The coefficient on lagged output for government spending is not significant, thus the government spending elasticity to output is assumed to have a normal distribution with mean -0.05. The transfers elasticity to output is assumed to have a mean of -0.1.

The parameter measuring the co-movement of innovations in capital and labor income tax rates is assumed to have a normal distribution with a mean of 0.25 and standard deviation of 0.1. This is keeping in line with Leeper, Plante and Traum (2010) who model similar correlation between the two tax rate shocks.

5 Multipliers Implied by the Priors

To evaluate the economic content of the priors of the parameters being estimated, Table 1 shows their implications for the fiscal multipliers, that are the focus of the paper. The table reports the median and 95 percentile present value multipliers for 500 random draws from the prior distribution of the parameters. Since deep habits are introduced as a transmission mechanism, notice that the median impact multiplier for government spending is larger than 1. However, as the confidence bands illustrate that the priors do not exclude the possibility of a much smaller spending multiplier. In general, tax multipliers are smaller than spending multiplier at early horizons. Also, note that the confidence bands are large, particularly for longer horizons, which reflects the disperse priors for fiscal rule parameters.

Table 1: Present Value Multipliers Implied by the Priors

	Quarter 1	Quarter 4	Quarter 12	Quarter 20
Government Spending Multiplier				
$\frac{PV\Delta Y_{t+k}}{PV\Delta G_{t+k}}$	1.06	0.93	0.41	0.12
	[0.7, 1.8]	[0.5,1.9]	[-0.2, 1.7]	[-0.65, 1.4]
Labor Tax Multiplier				
$\frac{PV\Delta Y_{t+k}}{PV\Delta T^w_{t+k}}$	0.10	0.22	0.40	0.25
	[0.0,0.3]	[0.0, 0.5]	[-1.0, 0.9]	[-6.2, 1.5]
Capital Tax Multiplier				
$\frac{PV\Delta Y_{t+k}}{PV\Delta T^k_{t+k}}$	0.45	0.61	0.73	0.59
	[0.3, 0.8]	[0.3, 1.1]	[-0.3, 1.6]	[-5.0, 1.9]

Note: This table shows the present discounted value of the cumulative change in output over the present value cumulative change in the fiscal variable of interest, over the k quarters, for 500 random draws from the prior distribution of the parameters. The reported numbers are the median multipliers and the 95 percentiles are given below in brackets.

6 Fit of the Model

In order to assess the goodness of fit of the model, Figure 1 shows the data used in the estimation, along with the posterior mean of the smoothed series implied by the estimated model. The fit of the model is nearly perfect for most variables, notably government spending and tax rates. The model predicts consumption relatively smoother than is observed. The only significant discrepancy is inflation where the model implies less overall volatility.

Table 2 also reports the standard deviations computed from data and those implied by the model. It also reports the 90 percent probability intervals that account for both parameter uncertainty and small sample uncertainty. Relative to the data, the model over-predicts the standard deviation of output a little, and approximately matches the relative standard deviation of consumption, inflation and hours. There is some tendency to over-predict the volatility of investment, and tax rates and under predict the volatility of nominal interest rate and government spending. Note that the estimated model does not perfectly match these moments, since I am employing a likelihood based estimation procedure, which tries to match the entire structure of the data series, including second moments, autocorrelations and cross-correlations.

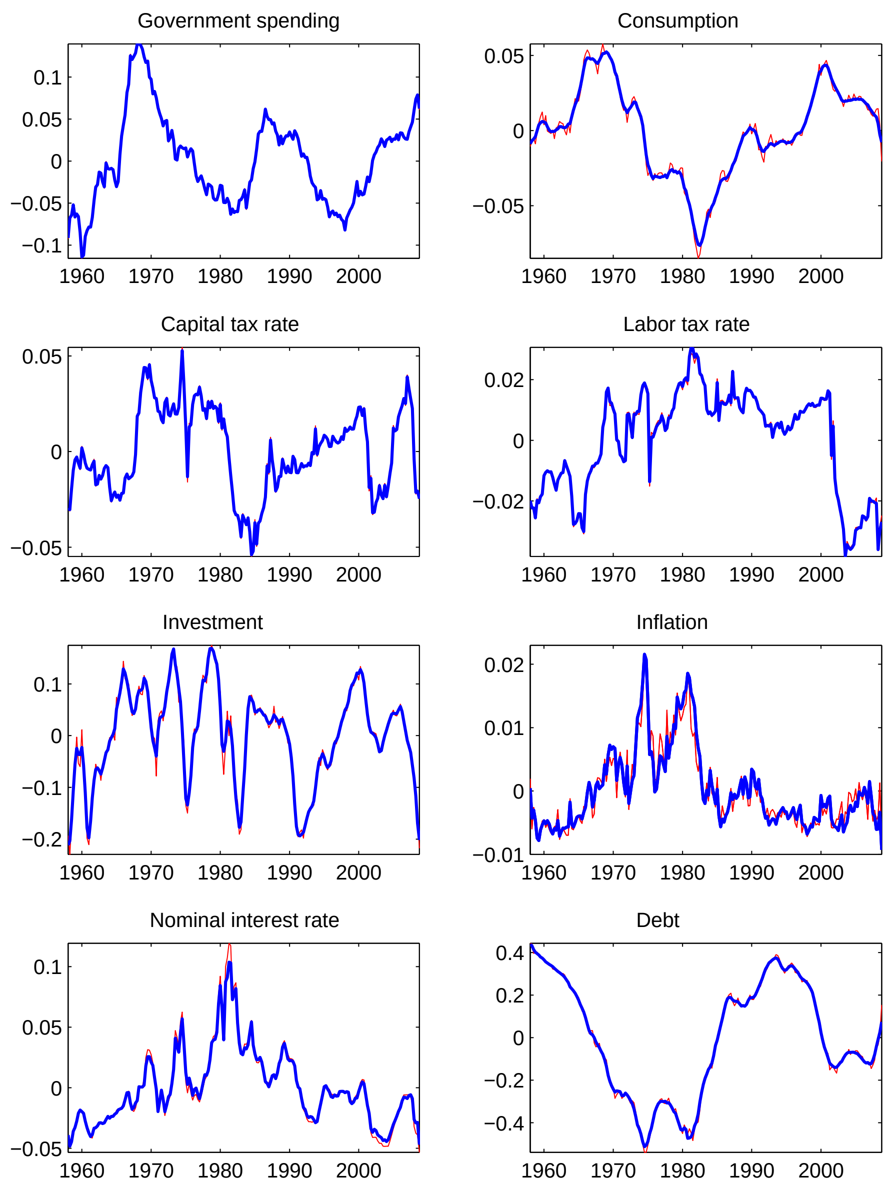
Figure 2 shows the autocorrelations and cross-correlations generated by the model and in the data for selected observable variables. The model predictions are the black lines, where the solid black line is the median and the dashed lines are the 90 percent posterior intervals. The data is represented by the grey lines. The diagonal of the figure shows that the model is able to capture the decaying autocorrelation structure of the variables quite well. Generally, the data cross-correlations fall within the confidence bands. These error bands, however, are quite large, accounting for both parameter and small sample uncertainty.

Table 2: Moment Comparison

	Data	Model	
		Median	[5,95]
Std. Dev. of Output (percent)	3.62	4.74	[3.24, 5.31]
Standard Deviation/ Standard Deviation of Output			
Consumption	0.83	0.70	[0.44, 1.04]
Investment	2.94	3.79	[2.61, 5.16]
Inflation	0.16	0.16	[0.11, 0.23]
Nominal Interest Rate	0.87	0.34	[0.22, 0.55]
Government Spending	1.41	1.18	[0.75, 1.76]
Capital tax rate	0.89	0.68	[0.37, 1.14]
Labor tax rate	0.45	0.36	[0.18, 0.66]
Hours	1.02	1.71	[1.32, 2.37]

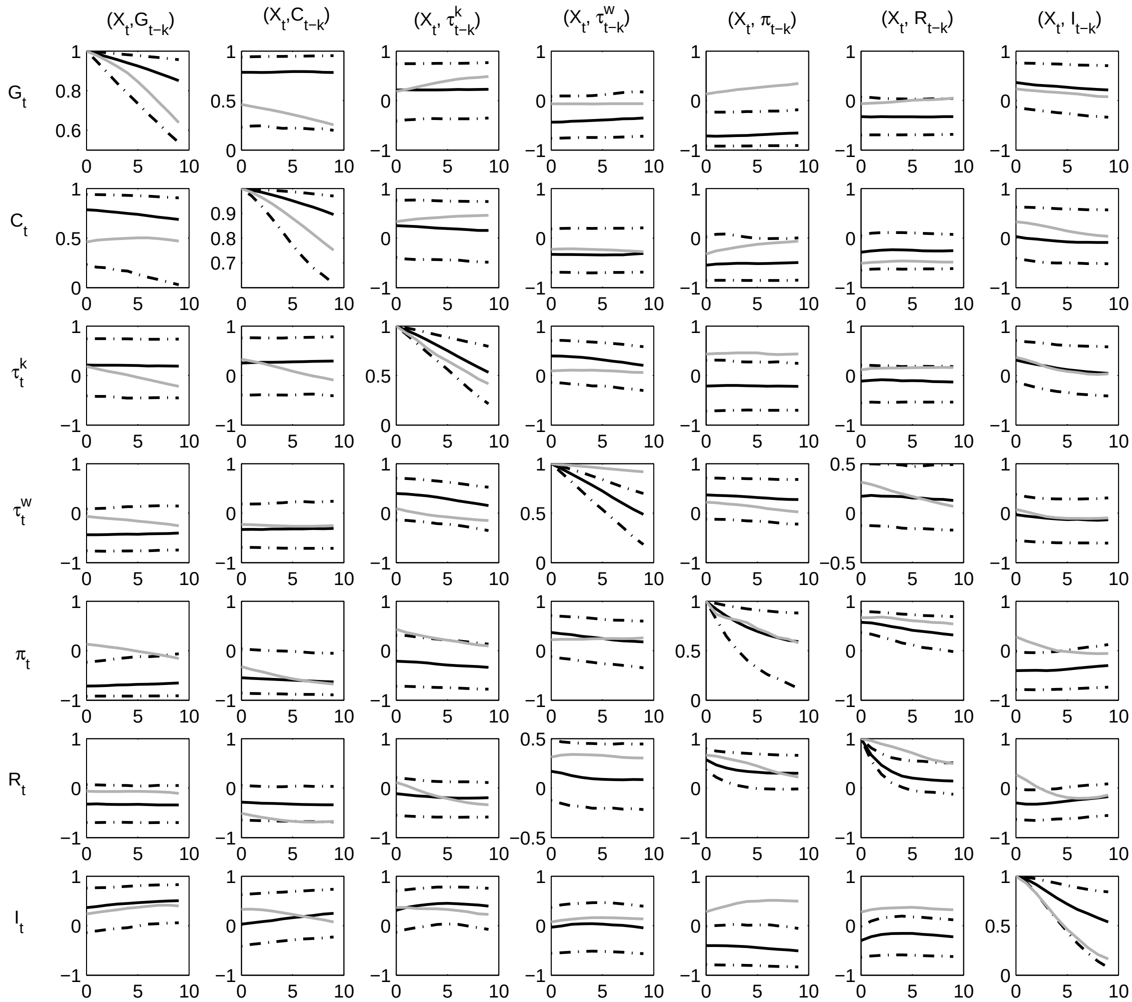
For randomly chosen 1000 draws, I generate 500 samples of the observable series implied by the model with the same length as the data-set (204 observations) after discarding the first 80 initial observations. The table reports the median and 5th and 95th percentile together with the corresponding moment in the data.

Figure 1: Model fit



Note: The thin red line is the data used in the estimation and the thick blue line is posterior mean of the smoothed version of the same series.

Figure 2: Cross-correlations



Note: The black line represent the median cross-correlations implied by the model along with the 90 percent confidence bands (dash-dotted line). The grey lines are the data cross-correlations. Each column gives the correlation between X_t and the variable specified, where X_t is given in each row. The x-axis gives the values of k.

Table 3: Impact Multipliers

	Quarter 1	Quarter 4	Quarter 12	Quarter 20
Government Spending Multiplier				
$\frac{\Delta Y_{t+k}}{\Delta G_t}$	1.07	0.81	0.16	-0.15
	[1.01, 1.13]	[0.64, 1.02]	[-0.03, 0.44]	[-0.28, 0.08]
Labor Tax Multiplier				
$\frac{\Delta Y_{t+k}}{\Delta T_t^w}$	0.13	0.36	0.25	0.07
	[0.09,0.18]	[0.26, 0.50]	[0.13, 0.41]	[0.03, 0.19]
Capital Tax Multiplier				
$\frac{\Delta Y_{t+k}}{\Delta T_t^k}$	0.34	0.36	0.11	-0.08
	[0.30, 0.37]	[0.27, 0.44]	[0.01, 0.22]	[-0.20, 0.02]

Note: These measure the increase in the level of output k quarters ahead in response to a change in the fiscal variable of interest at time t. The reported numbers are the median multipliers and the 95 percentiles are given below in brackets.

7 Estimated Impact Fiscal Multipliers

The stimulative effects of a fiscal action are generally framed in terms of multipliers. One measure is the impact multiplier which is defined as the increase in the level of output k periods ahead in response to a change in the fiscal variable of interest given by ΔF_t at time t.[2]

$$\text{Impact multiplier } k \text{ periods ahead} = \frac{\Delta Y_{t+k}}{\Delta F_t}.$$

So the spending impact multiplier is given by, $\frac{\Delta Y_{t+k}}{\Delta G_t}$, and for the tax rates the impact multiplier is given in terms of the change in total tax revenues, so its $\frac{\Delta Y_{t+k}}{\Delta T_t}$, where T_t denotes tax revenues. The two tax shocks are normalized so that they result in a 1 percent decrease in total tax revenues.

The median impact multipliers for the estimated model are reported in Table 3, along with 95 percentile confidence bands for horizons of 1, 4, 12 and 20 quarters after the shock hits the economy. The government spending multiplier for output is 1.07 on impact and slowly decreases to be negative in the long-run. This means that on impact, a 1 percent of GDP increase in government spending results in a larger than 1 percent overall increase in GDP.

The tax multipliers in the first quarter are small. A 1 percent of GDP fall in total tax revenues driven by labor tax cuts and capital tax cuts result in a 0.13 percent and 0.34 percent rise in GDP, respectively. But the effects of taxes take time to build, and both the capital and labor tax

[2]For instance the government spending multiplier is computed as follows, $\frac{\Delta Y_{t+k}}{\Delta G_t} = \frac{\%\Delta Y_{t+k}}{\%\Delta G_t}\frac{Y}{G}$, where Y and G are the steady state values of output and government spending respectively.

multipliers are maximized between 4 and 12 quarters. However, magnitude-wise taxes consistently have a smaller multiplier than spending for shorter horizons, and exceed the spending multiplier for horizons of 12 and 20 quarters.

The discussion in the paper focuses on present value multipliers, which are the same as the impact multipliers on impact, but additionally take into account the future path of the fiscal variables.

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