Not for Publication Appendix for: Interest Rate Rules and Equilibrium Stability under Deep Habits

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Abstract

This Appendix presents the determinacy results for the extended version of the model presented in Zubairy (2011) under various interest rate rules and robustness analysis. This extended model includes capital accumulation, a more generalized formulation of deep habits and government spending.

1 Extended Model with Deep Habits

In this section, the model is extended in several dimensions. I introduce capital accumulation, consider a more general formulation of deep habits and introduce government spending into the model. This framework is richer than the one considered in Section 2 of Zubairy (2011), and is close to the model considered in Ravn et. al (2007) and Zubairy (2009) where deep habits are considered as a transmission mechanism for demand shocks, namely government purchases shock.

1.1 Household

The economy is populated by a continuum of identical households of measure one indexed by $j \in [0,1]$. Each household $j \in [0,1]$ derives utility from consumption, x_t^c and disutility from labor supply, h_t and seeks to maximize lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(x_t^c, h_t) + V(x_t^g) \right\}. \tag{1}$$

Note, households also derive utility from consumption of public goods, x_t^g, and it is assumed to be separable from utility over private consumption and labor.[1] Unlike the simpler model, the stock of external habit is assumed to depend not just on consumption in the previous period but also on consumption in all the past periods. Thus, the variable x_t^c, the composite of habit adjusted consumption, is given by

$$x_t^c = \left[\int_0^1 (c_{it} - b^c s_{it}^C)^{1-\frac{1}{\eta}} di \right]^{1/(1-\frac{1}{\eta})}, \tag{2}$$

where s_{it} denotes the stock of habit in consuming good i in period t. Habits evolve over time according to the following law of motion,

$$s_{it+1}^C = \rho^c s_{it}^C + (1-\rho^c) c_{it}. \tag{3}$$

The parameter $\rho^c \in [0,1)$ measures the speed of adjustment of the stock of external habit to variations in the cross-sectional average level of consumption of variety i. When ρ^c takes the value zero, habit is measured by past consumption.

The household is also assumed to own physical capital, k_t, which accumulates according to the following law of motion,

$$k_{t+1} = (1-\delta) k_t + i_t, \tag{4}$$

where i_t denotes investment by the household and δ denotes the rate of depreciation of physical capital. Investment is assumed to be a composite good using intermediate goods in the following way, $i_t = \left[\int_0^1 (i_{it})^{1-\frac{1}{\eta}} di \right]^{1/(1-\frac{1}{\eta})}$ and the demand function for i_{it} is given as follows,

$$i_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} i_t. \tag{5}$$

[1]This means that the marginal utility for household consumption and labor are unaffected by the consumption of public goods.

1.2 Government

Like households, the government is also assumed to form habits over its consumption of individual varieties of goods. This can be thought of as households deriving utility from public goods that is separable from private consumption and leisure, and they exhibit good-by-good habit formation for these particular goods also. For instance, households care about the provision of individual public goods, such as trash removal or street lighting, in their own constituency versus others. The government allocates spending over individual varieties of goods, g_{it}, so as to maximize the quantity of composite good produced with the differentiated varieties of goods according to the relation,

$$x_t^g = \left[\int_0^1 (g_{it} - b^g s_{it}^G)^{1-1/\eta} \right]^{1/(1-1/\eta)}.$$

The variable s_{it}^G denotes the government's stock of habit in good i and is assumed to evolve as follows,

$$s_{it+1}^G = \rho^g s_{it}^G + (1-\rho^g) g_{it}. \tag{6}$$

The government's problem consists of choosing g_{it}, $i \in [0,1]$, so as to maximize x_t^g subject to the budget constraint $\int_0^1 P_{it} g_{it} di \leq P_t g_t$.The resulting demand function for each differentiated good $i \in [0,1]$ by the public sector is,

$$g_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} x_t^g + b^g s_{it-1}^G. \tag{7}$$

Lump sum taxes are assumed to balance out government spending expenditures each period. Real government expenditures, denoted by g_t are assumed to be exogenous, stochastic and follow the following univariate first-order autoregressive process,

$$\hat{g}_t = \tilde{\rho}_g \hat{g}_{t-1} + \epsilon_t^g, \tag{8}$$

where $\hat{g}_t$ is the log deviation of spending from its steady state.

1.3 Firms

Each variety of final goods is produced by a single firm in a monopolistically competitive environment. Each firm $i \in [0,1]$ produces output using the production technology given by $F(k_{it}, h_{it})$, using capital services, k_{it}, and labor services, h_{it} as factor inputs. Similar to the simple model, each firm i faces quadratic price adjustment costs and demand for c_{it}, i_{it} and g_{it}, given by (**??**), (5) and (7) respectively.

2 Equilibrium Dynamics

Calibration of parameters is discussed in Section 2.5. Some additional parameters specific to the full model with the introduction of investment and government spending are calibrated as follows. The depreciation rate, δ, is set at 0.025, which implies an annual rate of depreciation on capital equal to 10 percent. θ is set at 0.3, which corresponds to a steady state share of capital income roughly equal to 30%. The share of government spending in aggregate output is taken at 0.2. Also, it is assumed that the speed of adjustment of the stock of habit for all kinds of goods is equal, so $\rho = \rho^c = \rho^g$, and this parameter is calibrated to be 0.9, based on estimates in Ravn et. al (2006) and Zubairy (2009).[2] However, I will be conducting sensitivity analysis for varying degrees of price stickiness (α) and the rate at which the habit stock accumulates (ρ). For sake of simplicity, deep habits parameters for household consumption and public consumption goods are restricted to be the same, so $b = b^c = b^g$.

Table 1: Calibrated parameters

β	δ	θ	η	h	g/y	α	ρ
0.9902	0.025	0.3	6	0.5	0.2	17.5	0.9

2.1 Monetary Policy Rules and Indeterminacy

Figure 1 shows the determinacy region as I vary the deep habit parameter along with the inflation coefficient under a monetary rule responding only to inflation where all other parameters are calibrated at their baseline values. It is apparent that for the case of no deep habit formation, $b = 0$, or low values of the deep habit parameter, a unique equilibrium is guaranteed for $\alpha_\pi > 1$, but this is not the case for high degree of deep habit formation.

The size of the region of indeterminacy shrinks gradually as α_R is increased, as shown in Figure 2. This suggests that inertial rules are more desirable in order to render macroeconomic stability.

Next, nominal interest rate is allowed to respond to deviations of output from steady state, and once again increasing α_Y improves the region of determinacy. While there is a significant improvement between the case of no response to output deviations ($\alpha_Y = 0$) and the case of $\alpha_Y = 0.5$, the region of determinate equilibria are not affected much by considering $\alpha_Y = 1$ relative to $\alpha_Y = 0.5$. So, a response of nominal interest rate to economic activity is also a desirable feature for an interest rate rule to lead to determinacy.

2.2 Intuition for Indeterminate Equilibria and Impulse Response Analysis

Suppose households anticipate an increase in aggregate demand, without any shocks to fundamentals to justify it. This increase in demand would be accompanied by an increase in hours worked, lower markups due to deep habits, and high inflation as the firms adjust prices to get to their wanted markups. But an interest rate rule that has $\alpha_\pi > 1$, will generate high real interest rate along the adjustment path and imply lower consumption and investment relative to steady state. Thus it would not be possible to sustain a boom in demand, and so it is not consistent with rational expectations.

On the other hand, consider the case where the degree of deep habits is sufficiently high to allow multiple equilibria. The impulse response functions for such an expansionary sunspot shock are shown in Figure 4. Here the model is calibrated so the Taylor principle is satisfied, $\alpha_\pi = 1.5$, and the deep habit parameter, $b = 0.96$. Now even if the interest rate rule follows the Taylor principle, the higher degree of deep habits will drive the markups countercyclical to a greater extent. Note that the markup, say μ_t, is a wedge between marginal product of labor and the real wage, i.e. $F_h(k_t, h_t) = \mu_t w_t$. This high deep habit formation helps in driving the markup sufficiently down, so for any given level of wage, marginal product of labor falls, and so labor demand rises. This shift in the labor demand leads to a rise in real wages. The increase in wages causes the households to substitute away from leisure to consumption, and so consumption of households rises. In other

[2] Ravn et. al(2006) assume $\rho = \rho^c = \rho^g$, and estimate it be 0.85. Zubairy (2009) allows different values for ρ^c and ρ^g and estimates them to be 0.89 and 0.98, respectively.

words, in such a case the degree of deep habit formation leads to intra-temporal substitution effects working in opposition to the intertemporal substitution effects. At the same time, increased labor demand also causes marginal product of capital to rise. This higher expected rate of return on capital leads to more investment. This rise in both consumption and investment is an increase in realized demand, as anticipated by agents in the economy, thus leading to self-fulfilling expectations.

2.3 Robustness to parameter values

This section considers the robustness of results to different choice of parameter values, namely for the price stickiness parameter, α and the speed of adjustment of the habit stock, ρ. I consider the conditions for a determinate equilibrium under a monetary policy rule where nominal rate *only* responds to inflation.[3] Figure 5 shows what the determinacy region looks like in our baseline model with the price stickiness parameter, α increasing along the y-axis, the deep habit parameter, b along the x-axis and under the assumption $\alpha_\pi = 1.5$ and $\alpha_R = \alpha_Y = 0$. Four different cases are shown with varying values of ρ, the speed of adjustment of the habit stock. Looking at the figure it is clear that when there are no deep habits in the model, i.e. $b = 0$, a unique local equilibrium exists. However, the degree of deep habits plays a crucial role and for a combination of high values of the deep habit parameter and high degree of price stickiness, the economy runs into a region of indeterminacy even when nominal interest rate is adjusting more than one-for-one with inflation. Note in the right most lower panel, that even in the case of no price rigidities, for the case of $\rho = 0.9$, multiple equilibria exist in the case of high degree of deep habits.

Figure 5 reaffirms the fact that the Taylor principle is no longer a sufficient condition to ensure the existence of a local unique equilibrium in the case of strong nominal rigidities and high degree of deep habit formation.

2.4 Backward Looking Interest Rate Rule

In this section, a backward looking monetary rule is considered, where the nominal interest rate responds to past inflation. The motivation behind adopting this rule is related to the information available to the monetary authority at any given time. Backward looking rules have also been recommended in the literature by many others, for instance, Carlstrom and Fuerst (2000) and Benhabib et. al (2001).[4] The rule considered has the following form,

$$\hat{R}_t = \alpha_\pi \hat{\pi}_{t-1}.$$

Figure 6 shows the region of determinacy when the deep habit parameter, b is varied along with the inflation coefficient, α_π, in the backward looking rule, in a model where all other parameters are set at their baseline values. In this case, the equilibrium is unique for higher degree of deep habits, however for lower values of b, there is determinacy in the case where the monetary policy actively responds to past inflation so $\alpha_\pi > 1$, but no too actively, so that α_π that guarantees a unique local equilibrium is bounded above. Notice, that this exercise is analogous to Figure 1 except that in Figure 6 the interest rate responds to past inflation instead of current inflation. Just by comparison of the two figures, under a backward looking rule, the equilibrium is determinate for higher degrees of deep habits, and unlike the case where nominal interest rate responds to current inflation, it is not always determinate for lower values of the deep habit parameter, b. This

[3]This implies a monetary policy rule of the form $\hat{R}_t = \alpha_\pi \hat{\pi}_t$, and so $\alpha_R = \alpha_Y = 0$.

[4]Benhabib, Schmitt-Grohé and Uribe (2003) show however, that to guarantee global stability, the interest rate responding to past inflation is not sufficient, and the interest rate should also be set as a function of past interest rate.

happens because when nominal interest rate responds to past inflation, it is predetermined in any given period. Therefore, now in the region where there was determinacy with a contemporaneous rule, the number of predetermined variables exceeds the number of eigenvalues within the unit circle by one, and thus there is no local equilibrium in that parameter space. There are still, however, multiple equilibria for very high degree of deep habit when α_π is greater than but close to 1.

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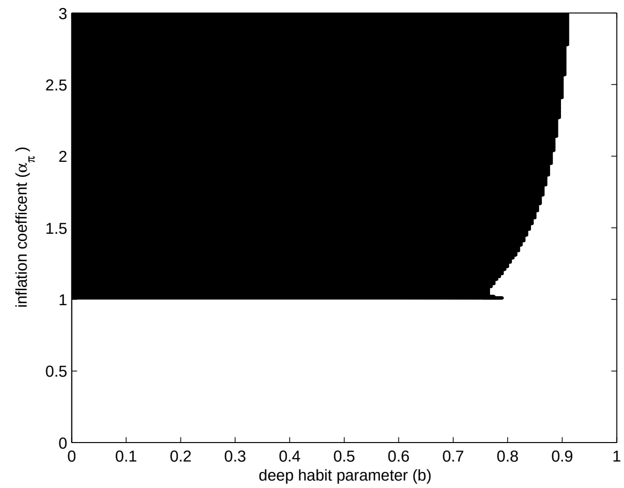

Figure 1: Region of determinacy under baseline calibration and a monetary rule that responds only to inflation with $\alpha_R = \alpha_Y = 0$.

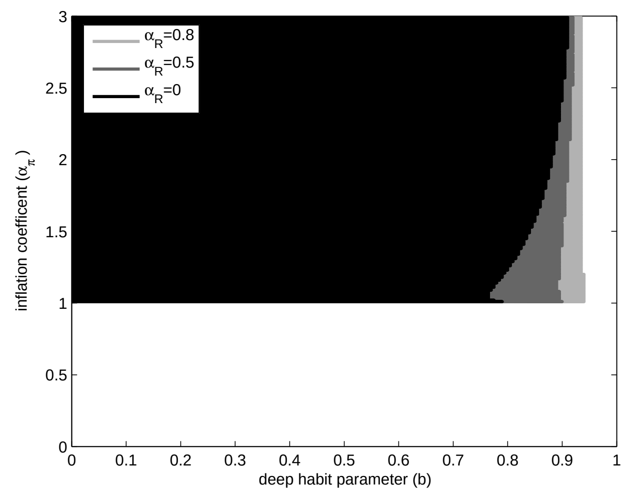

Figure 2: Regions of determinacy under baseline calibration, and $\alpha_Y = 0$.

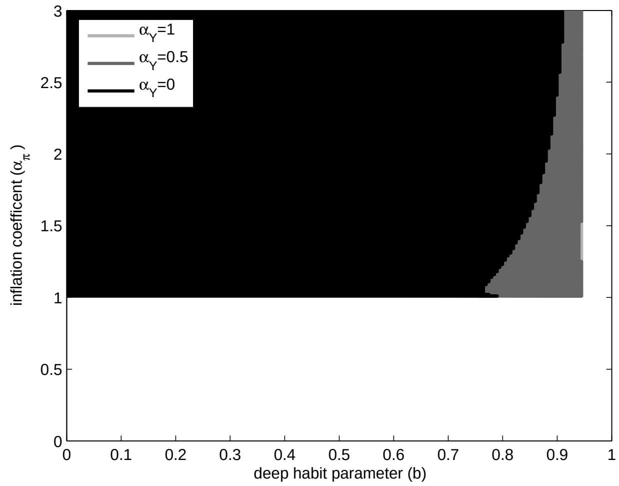

Figure 3: Regions of determinacy under baseline calibration, and $\alpha_R = 0$.

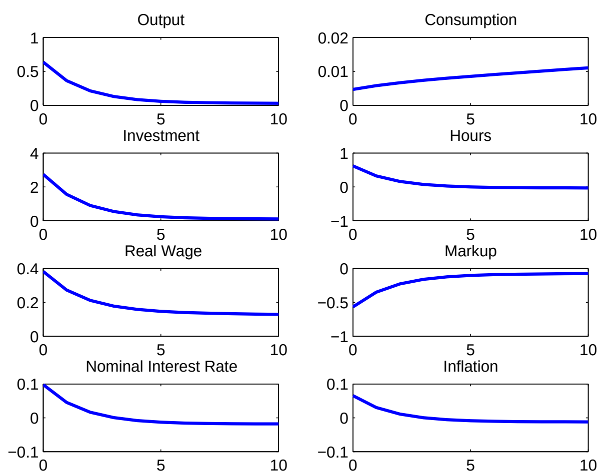

Figure 4: Response to a sunspot shock, where $b = 0.96$ and the monetary policy rule is given by $\hat{R}_t = 1.5\hat{\pi}_t$.

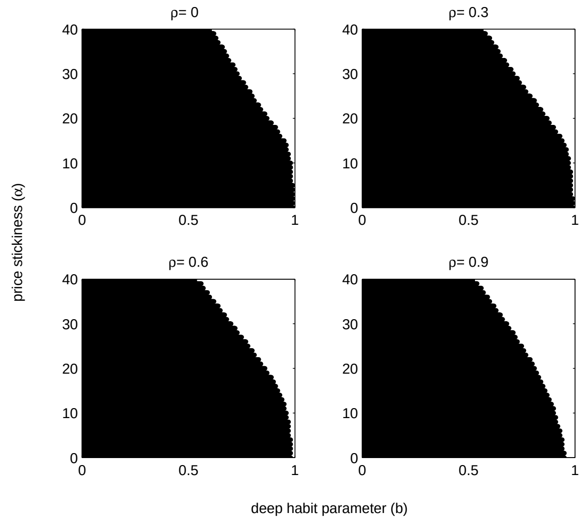

Figure 5: Region of determinacy in the baseline model for various values of the speed of adjustment of the habit stock (ρ), and a monetary policy rule with responding only to inflation with $\alpha_\pi = 1.5$ and $\alpha_R = \alpha_Y = 0$.

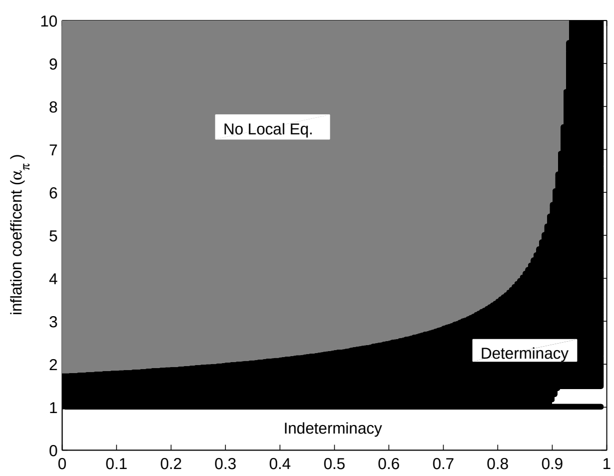

Figure 6: Region of determinacy under baseline calibration and a backward looking monetary policy rule of the form $\hat{R}_t = \alpha_\pi \hat{\pi}_{t-1}$.

APPENDIX

A Full Model With Deep Habits

A.1 Equilibrium Conditions

$$x_t^c = c_t - b^c s_t^C, \tag{B-1}$$

$$s_{t+1}^C = \rho^c s_t^C + (1-\rho^c)c_t. \tag{B-2}$$

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{B-3}$$

$$U_x(x_t^c, h_t) = \lambda_t, \tag{B-4}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t}{\pi_{t+1}}, \tag{B-5}$$

$$-U_h(x_t^c, h_t) = \lambda_t w_t, \tag{B-6}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1} \left[r_{t+1}^k + q_{t+1}(1-\delta) \right], \tag{B-7}$$

$$x_t^g = g_t - b^g s_t^g, \tag{B-8}$$

$$s_{t+1}^g = \rho^g s_t^g + (1-\rho^g)g_t, \tag{B-9}$$

$$\mathrm{mc}_t F_2(k_t, h_t) = w_t, \tag{B-10}$$

$$\mathrm{mc}_t F_1(k_t, h_t) = r_t^k, \tag{B-11}$$

$$\frac{1 - mc_t - \tilde{\nu}_t^c}{\rho^c - 1} = E_t r_{t,t+1} \pi_{t+1} \left[b^c \tilde{\nu}_{t+1}^c + \frac{\rho^c}{\rho^c - 1} \left\{ 1 - mc_{t+1} - \tilde{\nu}_{t+1}^c \right\} \right], \tag{B-12}$$

$$\frac{1 - mc_t - \tilde{\nu}_t^g}{\rho^g - 1} = E_t r_{t,t+1} \pi_{t+1} \left[b^g \tilde{\nu}_{t+1}^g + \frac{\rho^g}{\rho^g - 1} \left\{ 1 - mc_{t+1} - \tilde{\nu}_{t+1}^g \right\} \right], \tag{B-13}$$

$$1 - mc_t - \tilde{\nu}_t^i = 0, \tag{B-14}$$

$$\eta \left(\tilde{\nu}_t^c x_t^c + \tilde{\nu}_t^g x_t^g + \tilde{\nu}_t^i i_t \right) + \alpha \pi_t (\pi_t - 1) - (c_t + g_t + i_t) = \alpha \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1} (\pi_{t+1} - 1) \right], \tag{B-15}$$

$$F(k_t, h_t) = c_t + g_t + i_t + \frac{\alpha}{2}(\pi_t - 1)^2. \tag{B-16}$$