

Interest Rate Rules and Equilibrium Stability under Deep Habits

Sarah Zubairy*

Bank of Canada
234 Wellington St.
Ottawa, ON K1A 0G9
Phone: 613-782-8100
Fax: 613-782-7163
szubairy@bankofcanada.ca

Abstract

This paper studies determinacy of equilibrium in a new-Keynesian model with deep habits under different interest rate rules. The main finding is that an interest rate rule satisfying the Taylor principle is no longer a sufficient condition to guarantee determinacy. Including interest rate smoothing and a response to output deviations from steady state significantly enlarge the regions of determinacy. However, under all the simple interest rate rules considered, determinacy is not guaranteed for very high degree of deep habits. Deep habits give rise to counter-cyclical markups, which is in line with empirical evidence and make it an appealing feature in the study of demand shocks. The counter-cyclicalities of markups also lead to multiple equilibria due to self-fulfilling expectations for high degree of deep habit formation.

Keywords: Taylor principle, interest rate rules, sticky prices, deep habits

JEL Classification: E31, E52

*I would like to thank Stephanie Schmitt-Grohe, two anonymous referees, the associate editor and participants of the Duke University Macro Reading Group for useful comments. This is a substantially revised version of the paper previously circulated as “Deep Habits, Nominal Rigidities and Interest Rate Rules”. The views expressed in this paper are those of the author. No responsibility should be attributed to the Bank of Canada.

1 Introduction

An interest rate rule where the nominal interest rate adjusts by more than one-for-one in response to inflation is said to satisfy the Taylor principle. This is shown by Woodford (2001) to be a necessary and sufficient requirement to guarantee a locally unique rational expectations equilibrium in a standard new-Keynesian model. However, a number of papers have pointed out the limitation of the Taylor principle in avoiding indeterminacy and fluctuations driven by self-fulfilling fluctuations, when departing from standard modeling assumptions. These include among others Benhabib et. al (2001) and Carlstrom and Fuerst (2001) who consider different modeling choices for money, Galí et. al (2004) who consider a model with rule-of-thumb consumers and Sveen and Weinke (2005) who model firm-specific capital. The conditions for determinacy of a unique equilibrium are thus model-dependent, and so the robustness of simple interest rate rules to model specification is a concern.

In this paper, I show how introducing deep habits into a model affect the performance of simple interest rate rules, and assess whether the Taylor principle is a sufficient condition for determinacy. I analyze a standard new Keynesian model economy, and in this framework allow for households to exhibit deep habits, which is essentially external habit formation (or keeping up with the Joneses) on a good-by-good basis. Habit formation is a desirable feature in macroeconomic models since it helps account for the hump-shaped and persistent response of consumption to various shocks in the economy. Studying a model with deep habits, as introduced in Ravn et. al (2006), is of special interest since it is a more generalized version of habit formation, as agents form habits over consumption of individual goods that form the composite consumption good.

Deep habit formation give rise to the same consumption Euler equation, but unlike the more widely used habit formation at the level of a single aggregate good, they have additional consequences for the supply side of the economy. They render the firm's pricing problem dynamic, even in the absence of nominal rigidities, and give rise to time-varying markups of price over marginal cost. The implied counter-cyclical markups are consistent with the findings of the empirical literature (e.g. Rotemberg and Woodford (1999)), and additionally act as a transmission mechanism for the observed effects of demand shocks. For instance, deep habits with their implied counter-cyclical movements of markups have been shown to successfully explain the rise in wages and consumption in response to a government spending shock, an empirical observation that most standard model fail to predict (see Ravn et. al (2007) or Zubairy (2009)).

The main findings of this paper can be summarized as follows. In a model with deep habits, if the monetary authority follows a rule where the nominal interest rate strictly responds to current inflation, then the Taylor principle is too weak a condition to render stability. I also show that including interest rate smoothing and a response to output deviations from steady state into the monetary policy rule significantly enlarge the regions of determinacy. However, under all the simple interest rate rule considered here, when nominal interest rate responds to contemporaneous variables, determinacy is not always guaranteed for very high degrees of deep habits.

The economic intuition behind deep habits giving rise to sunspot equilibria is as follows: Suppose

agents in the economy expect higher demand. Due to deep habit formation in the model, a unit sold in the current period increases expected sales in the next period. This leads to firms lowering markups in order to hook a larger customer base to carry to the next period. Lowering markups cause firms to increase labor demand which in turn leads to a higher wage. Agents substitute from leisure to consumption as a result and consumption demand goes up. For high degree of deep habit formation, these positive effects on overall demand are large enough that expectations are self-fulfilling.

The remainder of the paper is as follows: Section 2 describes a simple model with deep habits, and how the introduction of deep habits affects the Phillips curve. Section 3 describes the equilibrium properties of the model and analyzes the conditions required for the determinacy of a local unique equilibrium under various simple interest rate rules. Section 4 examines the robustness of these results under different parameter values, extending the model to include capital and government, and habit formation at the level of a single aggregate good. And finally, Section 5 concludes.

2 Canonical New-Keynesian Model with Deep Habits

I am considering a model economy that features optimizing households and a continuum of profit maximizing firms producing intermediate goods. This is a canonical new-Keynesian model and the only departure is the presence of deep habit formation, or habit formation at the level of intermediate goods, for private consumption goods, as first introduced in Ravn et. al (2006).

2.1 Households

The economy is populated by a continuum of identical households of measure one indexed by $j \in [0, 1]$. Each household $j \in [0, 1]$ derives utility from consumption, x_t and dis-utility from labor supply, h_t and seeks to maximize lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^c, h_t). \quad (1)$$

The introduction of deep habits means that the agents do not form habits at the level of the aggregate consumption basket, given here by x_t^c , but at the level of individualized goods. This is then habit formation for a narrower category of goods. Thus, the variable x_t^c is a composite of habit adjusted consumption of a continuum of differentiated goods indexed by $i \in [0, 1]$,

$$x_t^c = \left[\int_0^1 (c_{it} - b^c c_{it-1})^{1-\frac{1}{\eta}} di \right]^{1/(1-\frac{1}{\eta})}. \quad (2)$$

In principle, households could exhibit a different degree of habit formation across the different individualized goods but for the sake of tractability, I assume it to be the same across the differentiated

goods. The parameter $b^c \in [0, 1)$ measures the degree of external habit formation, and when b^c is zero, the households do not exhibit deep habit formation. For any given level of consumption of x_t^c , purchases of each individual variety of goods $i \in [0, 1]$ in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the aggregation constraint (2), where P_{it} denotes the nominal price of a good of variety i at time t . The optimal level of demand, c_{it} for $i \in [0, 1]$ is then given by,

$$c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} x_t^c + b^c c_{it-1}, \quad (3)$$

where P_t is a nominal price index defined as $P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}$.

Note that consumption of each variety is decreasing in its relative price, P_{it}/P_t and increasing in the level of habit adjusted consumption, x_t^c . At the same time, the demand function has a second price-inelastic component given by $b^c c_{it-1}^C$. When there is an increase in aggregate demand, the price-elastic part gets a higher weight, which implies that price-elasticity is pro-cyclical and since markup is given by the inverse of the price-elasticity, it is counter-cyclical. Additionally, firms are forward-looking and internalize that the demand function has a backward looking term. When they expect high future demand, they have an additional incentive to lower their markups in order to appeal to a broader customer base and carry it over to the following period.

Households have access to a risk free nominal bond, B_t^j , that pays gross nominal interest rate R_t in period $t + 1$. They also pay lump-sum taxes in the amount T_t per period, and receive dividends from ownership of firms, ϕ_t . The household's period-by-period budget constraint is given by,

$$P_t x_t^j + \omega_t + B_t^j + T_t = R_{t-1} B_{t-1}^j + W_t h_t^j + \phi_t^j, \quad (4)$$

where $\omega_t = b^c \int_0^1 P_{it} c_{it-1} di$ and W_t is the nominal wage rate. The household chooses sequences for x_t^j , h_t^j and B_t^j so as to maximize the utility function (1) subject to (4), and a no-Ponzi game constraint.

The first-order conditions from the optimizing household's problem are,

$$\frac{-U_h(x_t^j, h_t^j)}{U_x(x_t^j, h_t^j)} = \frac{W_t}{P_t}, \quad (5)$$

$$U_x(x_t^j, h_t^j) = \beta R_t E_t U_x(x_{t+1}^j, h_{t+1}^j) \frac{P_t}{P_{t+1}}. \quad (6)$$

The first equation equates the marginal rate of substitution between consumption and leisure to the real wage, and the second equation is the standard Euler equation.

2.2 Firms

Each variety of final goods is produced by a single firm in a monopolistically competitive environment. Each firm $i \in [0, 1]$ produces output using labor services, h_{it} as factor input, with a production technology given by $F(h_{it})$. The firm is assumed to satisfy demand at the posted price.

Formally, $F(h_{it}) \geq c_{it}$.

The objective of the firm is to choose contingent plans for P_{it} and h_{it} in order to maximize the present discounted value of dividend payments, given by $E_0 \sum_{t=0}^{\infty} q_t P_t \phi_t^i$ where q_t is a pricing kernel determining the period-zero value of utility from one unit of a composite good in period t ,¹ and,

$$\phi_t^i = P_{it}c_{it} - W_t h_{it} - P_t \frac{\alpha}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2. \quad (7)$$

Note that sluggish price adjustment is introduced following Rotemberg (1982), by assuming that the firms incur a quadratic price adjustment cost for the good it produces, and the parameter α is the degree of price stickiness. This modeling choice of price stickiness produces qualitatively similar aggregate dynamics as pricing mechanism based on Calvo (1983).²

The firm's problem is to maximize profits and solves the following problem,

$$\begin{aligned} \mathcal{L} = & E_t \sum_{t=0}^{\infty} q_t \left\{ P_{it}c_{it} - W_t h_{it} - P_t \frac{\alpha}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2 \right. \\ & \left. + P_t mc_{it} [F(h_{it}) - c_{it}] + P_t \nu_{it} \left[\left(\frac{P_{it}}{P_t} \right)^{-\eta} x_t + b c_{it-1} - c_{it} \right] \right\}. \end{aligned}$$

The first order conditions w.r.t h_{it} , c_{it} and P_{it} are as follows,

$$W_t = P_t mc_{it} F_h(h_{it}), \quad (8)$$

$$\nu_{it} = \frac{P_{it}}{P_t} - mc_{it} + b E_t \nu_{it+1} \frac{q_{t+1}}{q_t} \frac{P_{t+1}}{P_t}, \quad (9)$$

$$c_{it} - \nu_{it} \eta \left(\frac{P_{it}}{P_t} \right)^{-\eta-1} x_t - \alpha \left(\frac{P_{it}}{P_{it-1}} - 1 \right) \frac{P_t}{P_{it-1}} + \alpha E_t \frac{q_{t+1}}{q_t} \frac{P_{t+1}}{P_t} \left(\frac{P_{it+1}}{P_{it}} - 1 \right) \frac{P_{it+1}}{P_{it}^2} \frac{P_t^2}{P_t} = 0. \quad (10)$$

Equation(8) implies that the markup of price over marginal cost is a wedge between the marginal product of labor and real wage.³ Equation (9) states that the value of selling an additional unit of a good, ν_{it} , is the sum of short-run profit of the sale and the future expected profits associated with it. Because of deep habits, a unit sold in the current period increases sales by an additional b units in the next period. Equation (10) equates the costs and benefits of a unit increase in relative price $\frac{P_{it}}{P_t}$. The first term is an increase in revenue followed by the cost in the form of a decline in demand that the price change induces and finally the loss from the price adjustment cost.

¹It follows from Equation (6) in the household's problem, $q_t P_t = \beta^t U_x(x_t, h_t)$

²The presence of deep habits alone makes the pricing problem dynamic and so additionally accounting for dynamics due to Calvo-style pricing makes aggregation non-trivial.

³The Lagrange multiplier on the constraint that output is determined by demand, is in fact the marginal cost.

2.3 Monetary policy rule

The log-linearized monetary policy rule is assumed to have the following form,

$$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1 - \alpha_R) (\alpha_\pi \hat{\pi}_t + \alpha_Y \hat{y}_t), \quad (11)$$

where $\alpha_R \geq 0$, $\alpha_\pi \geq 0$ and $\alpha_Y \geq 0$, and \hat{R}_t , $\hat{\pi}_t$ and \hat{y}_t represent nominal interest rate, inflation and output log deviations from respective steady states.

2.4 Phillips Curve under Deep Habits

The detailed derivation of the steady state and log-linearized equations are shown in the Appendix. In this simple model with deep habits, and under the assumption that the interest rate only responds to deviations of inflation from steady state (i.e. $\alpha_R = \alpha_Y = 0$), the log-linearized Phillips curve in terms of marginal cost is given as follows,

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{h}{\alpha} (\eta(1-b) - (1-b\beta)) \hat{m}c_t + b \frac{h}{\alpha} \left(\beta (E_t \hat{y}_{t+1} - \frac{1}{1-b} \hat{y}_t) - \frac{b\beta + 1}{(1-b)} (\hat{y}_t - \hat{y}_{t-1}) - \beta E_t \hat{\nu}_{t+1} \right). \quad (12)$$

Note that in the case of no deep habits, i.e. $b = 0$, this simplifies to the standard new-Keynesian Phillips curve. The presence of deep habits modifies the Phillips curve in several different ways. Firstly, the presence of deep habits affects the impact of marginal cost on inflation. Deep habits also introduce a backward looking term into the Phillips curve through the impact of the habit stock on the current period's demand, which would indicate that the inflation dynamics display more inertia. The presence of deep habits introduce additional forward looking terms in the form of $E_t \hat{\nu}_{t+1}$ and $E_t \hat{y}_{t+1}$. Notably, for a given path of inflation, an increase in expected future value of sales $E_t \hat{\nu}_{t+1} > 0$, reduces the markup which is $-\hat{m}c_t$. In addition, an increase in current demand, $\hat{y}_t > 0$ also reduces the markup.

3 Equilibrium Dynamics

3.1 Equilibrium Stability in the Model

As shown in the Appendix, the system of linearized equilibrium conditions can be reduced to four linear difference equations, namely the Phillips curve, the dynamic equation describing the final goods markup, an equation connecting past and current consumption and the Euler equation. The system of equation is given as follows:

$$\begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{\nu}_{t+1} \\ \hat{c}_t \\ \hat{c}_{t+1} \end{pmatrix} = C \begin{pmatrix} \hat{\pi}_t \\ \hat{\nu}_t \\ \hat{c}_{t-1} \\ \hat{c}_t \end{pmatrix}$$

The coefficients of the matrix C are functions of the steady state values and parameters of the model. This system can now be used to infer if the equilibrium is determinate by comparing the number of roots of the matrix C outside the unit circle relative to the number of non-predetermined variables. In this model, there is one pre-determined variable, c_{t-1} and three non-predetermined state variables, c_t , π_t and ν_t . I do not analytically derive necessary conditions for the existence of local equilibria because the analytical eigenvalues of the 4 by 4 non-sparse matrix C are too messy.

Instead, I present numerical results. The model is calibrated to quarterly frequency. The discount factor β , is set at $1.03^{-1/4}$, which implies a steady-state annualized real interest rate of 3 percent. Goods elasticity of substitution, η is set at 6, which implies a steady state markup of 20 percent in the absence of deep habits, consistent with average markup values discussed in Rotemberg and Woodford (1992). Also, the steady state labor, h is set at 0.5. In the baseline calibration, the price stickiness parameter, α , following Schmitt-Grohé and Uribe (2004), where they model price stickiness with a Rotemberg price adjustment cost, is set to be 17.5.⁴

Table 1: Calibrated parameters

β	η	h	α
0.9902	6	0.5	17.5

I characterize regions in the parameter space for which the equilibrium is determinate by computing the number of explosive eigenvalues of C for combinations of the monetary policy parameter, α_π and the deep habit parameter, b . A determinate equilibrium requires three explosive eigenvalues of the system. Figure 1 shows the determinacy region as I vary the deep habit parameter along with the inflation coefficient under a monetary rule responding only to inflation. Note that $\alpha_\pi > 1$ is a necessary condition for determinacy and for $\alpha_\pi < 1$ there are only two eigenvalues outside the unit circle and so the economy exhibits one degree of indeterminacy. For large values of deep habit formation parameter, $\alpha_\pi > 1$ is not a sufficient condition to guarantee a stable equilibrium and Figure 1 shows there is only one eigenvalue outside the unit circle in the right quadrant. To sum, this figure shows that the Taylor principle is no longer a sufficient condition to ensure the existence of a local unique equilibrium in the case of high degree of deep habit formation.

INSERT FIGURE 1 HERE

3.2 Intuition for Indeterminate Equilibria and Impulse Response Analysis

In the previous section, I show that for very high degree of deep habits, a unique equilibrium converging to a steady state does not exist. To get further insight into this finding, I consider the same baseline model where in the monetary policy rule the nominal interest rate only responds to

⁴This value of the price stickiness parameter implies that firms change their price on average every 3 quarters in a Calvo-Yun staggered-price setting model, based on estimates of a linear new-Keynesian Phillips curve by Sbordone (2002). Refer to a more detailed discussion in Schmitt-Grohé and Uribe (2004).

current inflation.⁵ Suppose households anticipate an increase in aggregate demand, without any shocks to fundamentals to justify it. This increase in demand would be accompanied by an increase in hours worked, lower markups due to deep habits, and high inflation as the firms adjust prices to get to their wanted markups. But an interest rate rule that has $\alpha_\pi > 1$, will generate high real interest rate along the adjustment path and imply lower consumption and investment relative to steady state. Thus it would not be possible to sustain a boom in demand, and so it is not consistent with rational expectations.

INSERT FIGURE 2 HERE

On the other hand, consider the case where the degree of deep habits is sufficiently high to allow multiple equilibria.⁶ The impulse response functions for such an expansionary sunspot shock are shown in Figure 2. Here the model is calibrated so the Taylor principle is satisfied, $\alpha_\pi = 1.5$, and the deep habit parameter, $b = 0.85$. Now even if the interest rate rule follows the Taylor principle, the higher degree of deep habits will drive the markups countercyclical to a greater extent. Note that the markup, say μ_t , is a wedge between marginal product of labor and the real wage, i.e. $F_h(h_t) = \mu_t w_t$. This high deep habit formation helps in driving the markup sufficiently down, so for any given level of wage, marginal product of labor falls, and so labor demand rises. This shift in the labor demand leads to a rise in real wages. The increase in wages causes the households to substitute away from leisure to consumption, and so consumption of households rises. In other words, in such a case the degree of deep habit formation leads to intra-temporal substitution effects working in opposition to the intertemporal substitution effects.⁷ This rise in consumption is an increase in realized demand, as anticipated by agents in the economy, thus leading to self-fulfilling expectations.

Schmitt-Grohe (1997) shows other models with variable countercyclical markups, in particular the implicit collusion model of Rotemberg and Woodford (1992) and a variant of Gali (1994) with increasing returns to scale, are also characterized by an indeterminate equilibrium for markup values in the upper range of empirical estimates.⁸ The economic intuition in those cases is also similar.

⁵In order to understand specifically the role of deep habits, I consider the case where there are no adjustment costs for prices, so $\alpha = 0$, and in this case the degree of indeterminacy is 1. See Section 4.1 for further details about the interaction of price stickiness and deep habits.

⁶Here I am considering the multiple equilibria characterized in the top right quadrant in Figure 1.

⁷Note also that if in fact, $\alpha > 0$ and there is price stickiness in the model, that further dampens the rise in inflation and the intertemporal substitution effects, and causes a further rise in demand.

⁸In context of the model of deep habits, the higher degree of habit formation raises the steady state markup value and so for instance in the example shown above, $b = 0.96$ corresponds to a steady state markup of 1.38 and is characterized by an indeterminate equilibrium. Schmitt-Grohe (1997) finds the minimum markup leading to sunspot equilibria is around 1.7, which is significantly higher but direct comparisons are difficult due to different calibrations of other parameters.

3.3 Monetary Policy Rules and Indeterminacy

Figure 1, as discussed in Section 3.3, shows the determinacy region as I vary the deep habit parameter along with the inflation coefficient under a monetary rule responding only to inflation. It is apparent that for the case of no deep habit formation, $b = 0$, or low values of the deep habit parameter, a unique equilibrium is guaranteed for $\alpha_\pi > 1$. Woodford (2001) shows that in the case of a simple new-Keynesian model, when there is a zero coefficient on the output gap in the monetary policy rule, namely $\alpha_Y = 0$, then $\alpha_\pi > 1$, satisfies the Taylor principle and guarantees the existence of a local unique equilibrium. Under deep habits, the Taylor principle is no longer a sufficient condition to ensure the existence of a local unique equilibrium in the case of high degree of deep habit formation. The next question that arises is if the region of determinacy can be enlarged by modifying the monetary policy rule. So far, only the case of $\alpha_\pi > 0$ has been considered, where $\alpha_R = \alpha_Y = 0$ in the monetary policy rule. Next, I formally analyze variation in these other policy rule coefficients.

INSERT FIGURE 3 HERE

In a standard new-Keynesian model with interest rate smoothing, the Taylor principle implies that monetary policy should be active in the long-run. So the particular value of α_R , the interest rate smoothing parameter, is irrelevant for determinacy, as long as $\alpha_\pi > 1$. I find, however, that with the introduction of deep habits, determinacy is not guaranteed for all $\alpha_R \in (0,1)$, even when $\alpha_\pi > 1$. In fact, the size of the region of indeterminacy shrinks gradually as α_R is increased, as shown in Figure 3. This suggests that inertial rules are more desirable in order to render macroeconomic stability. Next, nominal interest rate is allowed to respond to deviations of output from steady state, and once again increasing α_Y widens the region of determinacy. As shown in Figure 4, there is a significant enlarging of the determinacy region between the case of no response to output deviations ($\alpha_Y = 0$) and the case of $\alpha_Y = 0.5$. For the case of $\alpha_Y = 1$, the region of determinate equilibria now also includes very high degree of deep habit formation when there is a small response to inflation. So, a response of nominal interest rate to economic activity is also a desirable feature for an interest rate rule to lead to determinacy.

INSERT FIGURE 4 HERE

The finding that combining active monetary policy with interest rate smoothing and responsiveness of nominal interest rate to economic activity improves the determinacy properties of the model is common across significantly different models.⁹ Note, however, that allowing for interest rate smoothing and/or response to economic activity still gives us indeterminacy for very large degrees of deep habits. The estimates for deep habit parameter in the context of medium-scale dynamic general equilibrium models as well as simpler frameworks similar to the one considered

⁹Among others, Gali et. al (2004) and Sveen and Weinke (2005). Sveen and Weinke (2005) have output gap in the policy rule, which is the difference between output and its natural level (level of output absent any nominal rigidities).

here, are usually between 0.6 and 0.9 (See Ravn et. al (2006) and Zubairy (2009)). The equilibrium is determinate for these values of the deep habit parameter under some of the calibrations for α_R and α_Y considered here.

4 Robustness Analysis

4.1 Robustness to parameter values

INSERT FIGURE 5 HERE

This section considers the robustness of results to different a choice of parameter values for the price stickiness parameter, α . Figure 5 shows what the determinacy region looks like in the baseline model with the price stickiness parameter, α increasing along the y-axis, the deep habit parameter, b along the x-axis and under the assumption $\alpha_\pi = 1.5$ and $\alpha_R = \alpha_Y = 0$. Looking at the figure it is clear that when there are no deep habits in the model, i.e. $b = 0$, a unique local equilibrium exists for all values of α . However, the degree of deep habit formation plays a crucial role and for high values of the deep habit parameter, the economy runs into a region of indeterminacy even though nominal interest rate is adjusting more than one-for-one with inflation. Notice also, that when there is no price stickiness (i.e. $\alpha = 0$) or for very low values of α , the model still has multiple equilibria for high degrees of habit formation, and the indeterminacy is of degree one. For higher values of α and deep habit formation, the degree of indeterminacy is 2. To get some intuition behind this, consider Equation (9) and (10) in the firm's problem, where it is clear that in addition to the presence of deep habits, price stickiness also affect markup dynamics. Price stickiness, measured by α , cause firms to smooth price increase over time in response to changes in marginal costs or aggregate demand. Thus both mechanisms amplify the effects of shocks to the economy, and for high degree of price stickiness along with deep habit formation, lead to equilibrium indeterminacy.

4.2 Model of Deep Habits with Capital and Government Spending

In this section, I consider the effects of extending the model presented in Section 2 in several dimensions. I introduce capital accumulation, consider a more general formulation of deep habits and introduce government spending into the model. This framework is close to the model considered in Ravn et. al (2007) and Zubairy (2009) where deep habits are considered as a transmission mechanism for demand shocks, namely government purchases shock. The details of the model are given in the Not-For-Publication Appendix (Zubairy, 2011).

INSERT FIGURE 6 HERE

Figure 6 shows the determinacy region for this extended model as I vary the deep habit parameter along with the inflation coefficient under a monetary rule responding only to inflation. It is apparent that for the case of no deep habit formation, $b = 0$, or low values of the deep habit parameter, a unique equilibrium is guaranteed for $\alpha_\pi > 1$, but this is not the case for high degree of

deep habit formation. Namely, $\alpha_\pi > 1$ while a necessary condition, is no longer sufficient to ensure a determinate equilibrium.¹⁰ Specifically, the response to inflation required in order to guarantee a determinate equilibrium is increasing in the degree of deep habit.

4.3 Habit Formation Over a Single Aggregate Good

In this section consider the more standard form of habit formation, where agents form habits over the composite consumption good, and each household maximizes its utility function,

$$U(c_t, h_t) = \frac{\left[(c_t - \theta \tilde{c}_{t-1})^{1-\nu} (1-h)^\nu \right]^{1-\sigma} - 1}{1-\sigma},$$

where $c_t = \left[\int_0^1 c_{it}^{1-\frac{1}{\eta}} \right]^{\frac{1}{1-\frac{1}{\eta}}}$. The parameter $\theta \in [0, 1)$ measures the degree of external habit formation, and \tilde{c}_{t-1} is the average consumption last period. The demand function for good i for a household in this case is given by, $c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} c_t$.

INSERT FIGURE 7 HERE

This specification of external habit formation is the same as commonly found in the literature, in order to match the persistence in the consumption response to macroeconomic shocks. Figure 7 shows the region of determinacy the model with superficial habits in the New-Keynesian model. Note that the Taylor principle is a necessary and sufficient condition to guarantee determinacy in this framework. The indeterminacy region in Figure 1, for $\alpha_\pi > 1$, is therefore precisely due to how deep habits affect the firm's problem and give rise to countercyclical markups, and similar results do not hold for habit formation at the level of the a single aggregate good, which only affects the demand side.

5 Conclusion

This paper shows how introducing deep habits into a model affects the performance of simple interest rate rules, where the nominal interest rate responds to inflation, output or is subject to interest-rate smoothing. The results suggest that the Taylor principle is too weak a condition to guarantee stability. In this standard new-Keynesian model with deep habits, including interest rate smoothing and a response to output deviations from steady state into the interest rate rule significantly enlarge the regions of determinacy. But, under all these rules, the equilibrium is not uniquely determined for very high degree of deep habits. The main intuition behind this finding is that at very high degrees of deep habits, the markups generated are large and countercyclical with resulting effects on consumption and investment demand that counter otherwise stabilizing effects of changes in real interest rate due to the monetary policy rule.

¹⁰Not-For-Publication Appendix (Zubairy, 2011) also explores the determinacy results for this extended model under different different interest rate rules, and robustness to parameter values.

This paper adds to the literature that suggests that the recommendations for monetary rules that render unique equilibrium are model dependent. It is then important to be more careful and aware of these problems of indeterminacy when augmenting models with new features.

References

- [1] Benhabib, Jess, Stephanie Schmitt-Grohé and Martin Uribe (2001), Monetary Policy and Multiple Equilibria. *American Economic Review*, 91(1), 167-185.
- [2] Calvo, Guillermo (1983), Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics*, 12, 383-398.
- [3] Carlstrom, Charles and Timothy Fuerst (2001), Timing and Real Indeterminacy in Monetary Models. *Journal of Monetary Economics*, 47(2), 285-98.
- [4] Gali, Jordi (1994), Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand, *Journal of Economic Theory*, 63(1), 73-96.
- [5] Gali, Jordi, David Lopez-Salido and Javier Valles (2004), Rule-of-Thumb Consumers and the Design of Interest Rate Rules. *Journal of Money, Credit and Banking*, 36(4), 739-763.
- [6] Ravn, Morten, Stephanie Schmitt-Grohé and Martin Uribe (2006), Deep Habits. *Review of Economic Studies*, 73, 195-218.
- [7] Ravn, Morten, Stephanie Schmitt-Grohé and Martin Uribe (2007), Explaining the Effects of Government Spending Shocks on Consumption and the Real Exchange Rate. Manuscript, Duke University.
- [8] Rotemberg, Julio (1982), Sticky Prices in the United States. *Journal of Political Economy*, 90(6), 1187-1211.
- [9] Rotemberg, Julio and Michael Woodford (1992), Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity. *Journal of Political Economy*, 100(6), 1153-1207.
- [10] Rotemberg, Julio and Michael Woodford (1999), The Cyclical Behavior of Prices and Costs. *Handbook of Macroeconomics*, 1051-1135.
- [11] Sbordone, Argia (2002), Prices and Unit Labor Costs: A New Test of Price Stickiness. *Journal of Monetary Economics*, 49, 265-292
- [12] Schmitt-Grohé, Stephanie (1997), Comparing Four Models of Aggregate Fluctuations due to Self-Fulfilling Expectations. *Journal of Economic Theory*, 72(1), 96-147.
- [13] Schmitt-Grohé, Stephanie and Martin Uribe (2004), Optimal Fiscal and Monetary Policy under Sticky Prices. *Journal of Economic Theory*, 114, 198-230.
- [14] Sveen, Tommy and Lutz Weinke (2005), New Perspectives on Capital, Sticky Prices and the Taylor Principle. *Journal of Economic Theory*, 123, 21-39.
- [15] Woodford, Michael (2001), The Taylor Rule and Optimal Monetary Policy. *American Economic Review*, 91, 232-237.

- [16] Zubairy, Sarah (2009), Explaining the Effects of Government Spending Shocks. Mimeo, Bank of Canada.
- [17] Zubairy, Sarah (2011), Not-for-Publication Appendix to: Interest Rate Rules and Equilibrium Stability under Deep Habits. Mimeo, Bank of Canada.

FIGURES

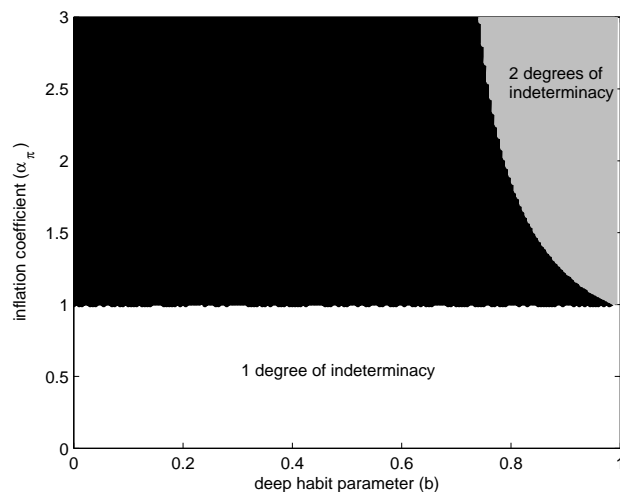


Figure 1: Region of determinacy under baseline calibration and a monetary rule that responds only to inflation with $\alpha_R = \alpha_Y = 0$. The equilibrium is determinate in the black region, and the white and gray regions corresponds to regions of indeterminacy.

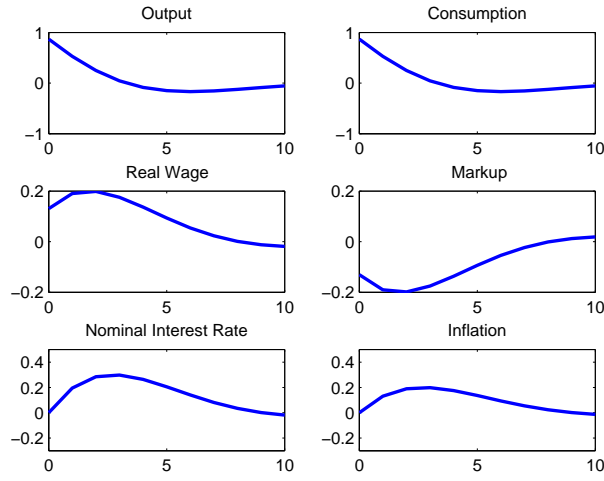


Figure 2: Response to a sunspot shock, where $b = 0.85$, $\alpha = 0$ and the monetary policy rule is given by $\hat{R}_t = 1.5\hat{\pi}_t$.

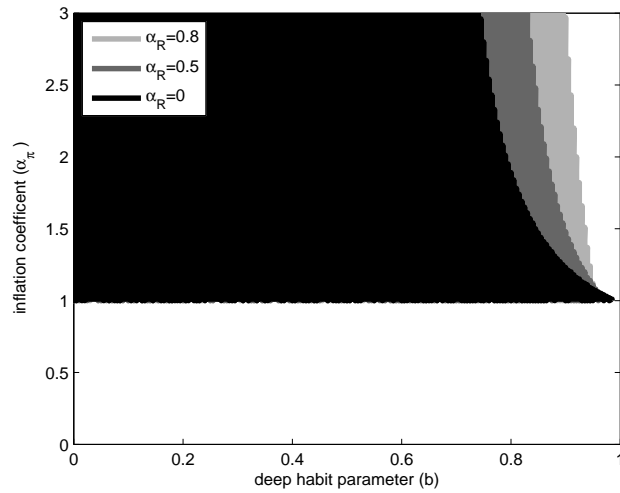


Figure 3: Regions of determinacy under varying degree of interest rate smoothing parameter, α_R , in the monetary policy rule and $\alpha_Y = 0$.

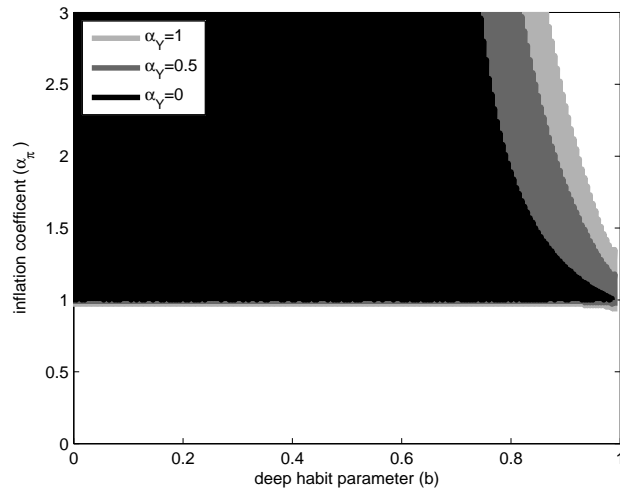


Figure 4: Regions of determinacy under varying degree of response to output gap, α_Y , in the monetary policy rule and $\alpha_R = 0$.

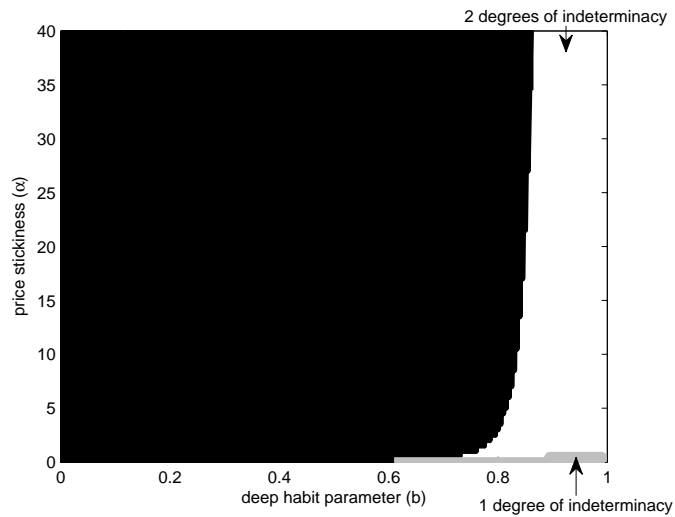


Figure 5: Robustness to degree of price stickiness: Region of determinacy for varying degree of price stickiness, α , under a monetary policy rule responding only to inflation with $\alpha_\pi = 1.5$ and $\alpha_R = \alpha_Y = 0$. The equilibrium is determinate in the black region, and the white and gray regions corresponds to regions of indeterminacy.

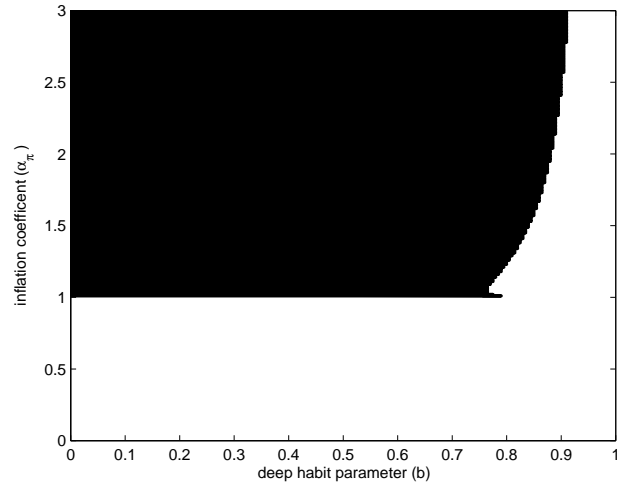


Figure 6: Robustness in extended model: Region of determinacy for the extended model with capital and government spending, and a monetary rule responding only to inflation with $\alpha_R = \alpha_Y = 0$.

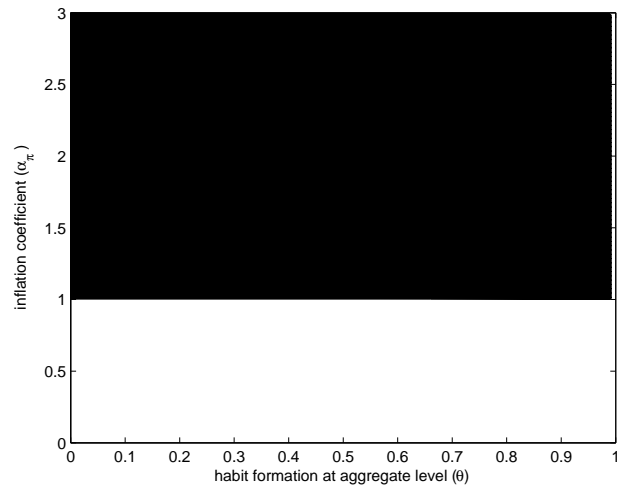


Figure 7: Robustness to habit formation over aggregate good: Region of determinacy in a model with nominal rigidities and superficial habits, and a monetary rule that responds only to inflation with $\alpha_R = \alpha_Y = 0$.

APPENDIX

A New Keynesian Model With Deep Habits

A.1 Equilibrium Conditions with Functional Forms

Note that the functional form for utility function is, $U(x_t, h_t) = (1 - \phi)\log(x_t) + \phi\log(1 - h_t)$, and we assume a linear production function so that $F(h_t) = h_t$. All households are identical so the consumption and labor supply across them is invariant. Additionally, since we consider a symmetric equilibrium, all firms charge the same price. Therefore, the following conditions characterize the equilibrium,

$$\frac{\phi x_t}{(1 - \phi)(1 - h_t)} = w_t \quad (\text{A-1})$$

$$\frac{1}{R_t} \frac{1}{x_t} = \beta E_t \frac{1}{x_{t+1} \pi_{t+1}} \quad (\text{A-2})$$

$$x_t = c_t - b c_{t-1} \quad (\text{A-3})$$

$$w_t = m c_t \quad (\text{A-4})$$

$$1 - m c_t - \nu_t + \beta b E_t \nu_{t+1} \frac{x_t}{x_{t+1}} = 0 \quad (\text{A-5})$$

$$c_t - \nu_t \eta x_t - \alpha (\pi_t - 1) \pi_t + E_t \alpha \beta \frac{x_t}{x_{t+1}} (\pi_{t+1} - 1) \pi_{t+1} = 0 \quad (\text{A-6})$$

and the resource constraint,

$$c_t + \frac{\alpha}{2} (\pi_t - 1)^2 = h_t \quad (\text{A-7})$$

A.2 Steady State

We pin down the following: $\pi = 1$ and $h = 0.5$, and the remaining steady state values are given as follows,

$$c = h, \quad x = (1 - b)c$$

$$R = \frac{1}{\beta}$$

$$\nu = \frac{1}{\eta(1 - b)}$$

$$m c = 1 + \nu(\beta b - 1), \quad w = m c$$

$$\phi = \frac{m c (1 - h)}{h(1 - b) + m c (1 - h)}$$

A.3 Log-linearized equations

Log linearizing the equilibrium conditions around the steady state yields the following:

$$\hat{x}_t = \frac{1}{1 - b} (\hat{c}_t - b \hat{c}_{t-1})$$

$$\hat{w}_t = \hat{m} c_t$$

$$\hat{h}_t = \hat{c}_t$$

$$\hat{m}c_t = \left(\frac{1}{(1-b)} + \frac{h}{1-h} \right) \hat{c}_t - \frac{b}{1-b} \hat{c}_{t-1}.$$

Also, the monetary policy rule is given by the following, so we ignore interest rate smoothing and response to output deviations from steady state for now,

$$\hat{R}_t = \alpha_\pi \hat{\pi}_t.$$

We substitute these expressions in the log-linearized equations that follow,

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \frac{h}{\alpha} \hat{v}_t + \frac{h}{\alpha} \frac{b}{(1-b)} \hat{c}_{t-1} - \frac{h}{\alpha} \frac{b}{(1-b)} \hat{c}_t$$

$$\frac{1}{1-b} (\hat{c}_{t+1} - (1+b)\hat{c}_t + b\hat{c}_{t-1}) = \alpha_\pi \hat{\pi}_t - E_t \hat{\pi}_{t+1}$$

$$E_t \hat{v}_{t+1} = \frac{1}{b\beta} \hat{v}_t + \frac{1}{1-b} E_t \hat{c}_{t+1}$$

$$+ \frac{\eta(1-b) + (b\beta - 1)}{b\beta} \left(\left(\frac{1}{(1-b)} + \frac{h}{1-h} \right) \hat{c}_t - \frac{b}{1-b} \hat{c}_{t-1} \right) - \frac{1+b}{1-b} \hat{c}_t + \frac{b}{1-b} \hat{c}_{t-1}$$

A.4 Determinacy of equilibrium

The equilibrium conditions can be written as follows, assuming perfect foresight,

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1}{1-b} \\ 0 & 0 & 1 & 0 \\ (1-b) & 0 & 0 & 1 \end{pmatrix}}_A \begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{v}_{t+1} \\ \hat{c}_t \\ \hat{c}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{\beta} & \frac{h}{\alpha\beta} & -\frac{h}{\alpha\beta} \frac{b}{1-b} & \frac{h}{\alpha\beta} \frac{b}{1-b} \\ 0 & \frac{1}{\beta b} & \frac{b}{1-b} (1-X) & X \left(\frac{1-bh}{(1-b)(1-h)} \right) - \frac{1+b}{1-b} \\ 0 & 0 & 0 & 1 \\ \alpha_\pi(1-b) & 0 & -b & (1+b) \end{pmatrix}}_B \begin{pmatrix} \hat{\pi}_t \\ \hat{v}_t \\ \hat{c}_{t-1} \\ \hat{c}_t \end{pmatrix}$$

where $X = \frac{\eta(1-b) + (b\beta - 1)}{b\beta}$. This can be further simplified, to write,

$$\begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{v}_{t+1} \\ \hat{c}_t \\ \hat{c}_{t+1} \end{pmatrix} = A^{-1} B \begin{pmatrix} \hat{\pi}_t \\ \hat{v}_t \\ \hat{c}_{t-1} \\ \hat{c}_t \end{pmatrix} = C \begin{pmatrix} \hat{\pi}_t \\ \hat{v}_t \\ \hat{c}_{t-1} \\ \hat{c}_t \end{pmatrix}$$

where

$$C = \begin{pmatrix} \frac{1}{\beta} & \frac{h}{\beta\alpha} & -\frac{bh}{(1-b)\beta\alpha} & \frac{bh}{(1-b)\beta\alpha} \\ -\frac{1}{\beta} + \alpha_\pi & \frac{1}{b\beta} - \frac{h}{\beta\alpha} & \frac{bh}{\alpha\beta} + \frac{b - ((1-b)\eta + b\beta - 1)\beta}{1-b} - \frac{b}{1-b} & \frac{(1-bh)\beta((1-b)\eta + b\beta - 1)}{b(1-b)(1-h)} - \frac{bh}{(1-b)\beta\alpha} \\ 0 & 0 & 0 & 1 \\ (1-b)\alpha_\pi - \frac{(1-b)}{\beta} & \frac{-h(1-b)}{\beta\alpha} & \frac{bh}{\beta\alpha} - b & 1 + b - \frac{bh}{\beta\alpha} \end{pmatrix}.$$

We now use this system to infer if the equilibrium is determinate by comparing the number of roots of the matrix C outside the unit circle relative to the number of non-predetermined variables which in this case are three.