Appendix for "Housing and Fiscal Policy"

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1 Equilibrium conditions of the model

1.1 Patient households

Consumption:

$$\frac{1}{c_{P,t}} = (1+\tau_c)\,\lambda_{P,t} \tag{1}$$

Owner-occupied housing:

$$\begin{aligned}&\left[1+\kappa_h\left(\frac{h_{P,t}}{h_{P,t-1}}-1\right)\frac{h_{P,t}}{h_{P,t-1}}\right]q_{h,t}\\ &\quad=\beta_P E_t\left[\frac{\xi_h}{\lambda_{P,t}h_{P,t}}+\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\left\{\begin{array}{c}\left[1-\delta_h-\tau_{p,t+1}(1-\tau_y)+\kappa_h\left(\frac{h_{P,t+1}}{h_{P,t}}-1\right)\left(\frac{h_{P,t+1}}{h_{P,t}}\right)^2\right]q_{h,t+1}\\ -I_{r,t+1}\tau_y\left(r_{h,t+1}-\widetilde{\delta}_{h,t+1}\right)\end{array}\right\}\right]\end{aligned} \tag{2}$$

Rental housing:

$$\begin{aligned}&\left[1+\kappa_h\left(\frac{h_{R,t}}{h_{R,t-1}}-1\right)\frac{h_{R,t}}{h_{R,t-1}}\right]q_{h,t}\\ &\quad=\beta_P E_t\left[\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\left\{\begin{array}{c}\left[1-\delta_h-\tau_{p,t+1}(1-\tau_y)+\kappa_h\left(\frac{h_{R,t+1}}{h_{R,t}}-1\right)\left(\frac{h_{R,t+1}}{h_{R,t}}\right)^2\right]q_{h,t+1}\\ +(1-\tau_y)\,r_{h,t+1}+\tau_y\widetilde{\delta}_{h,t+1}\end{array}\right\}\right]\end{aligned} \tag{3}$$

Capital:

$$q_{k,t} = \beta_P E_t\left[\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\left[(1-\delta_k)\,q_{k,t+1}+(1-\tau_k)\,r_{k,t+1}+\tau_k\delta_k\right]\right] \tag{4}$$

Labor:

$$\xi_l l_{P,t}^{\vartheta} = \lambda_{P,t}\,(1-\tau_y)\,w_{P,t} \tag{5}$$

Government bonds/lending:

$$1 = \beta_P E_t\left[\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\left[1+(1-\tau_b)\,r_t\right]\right] \tag{6}$$

1.2 Impatient households

Consumption:

$$\frac{1}{c_{I,t}} = (1+\tau_c)\,\lambda_{I,t} \tag{7}$$

Housing:

$$\begin{aligned}
&\left[1+\kappa_h\left(\frac{h_{I,t}}{h_{I,t-1}}-1\right)\frac{h_{I,t}}{h_{I,t-1}}-\mu_t\left(1-\rho_b\right)\phi\right]q_{h,t} \\
&\quad=\beta_I E_t\left[\frac{\xi_h}{\lambda_{I,t}h_{I,t}}+\frac{\lambda_{I,t+1}}{\lambda_{I,t}}\left\{\begin{array}{c}\left[1-\delta_h-\tau_{p,t+1}\left(1-\tau_y\right)+\kappa_h\left(\frac{h_{I,t+1}}{h_{I,t}}-1\right)\left(\frac{h_{I,t+1}}{h_{I,t}}\right)^2\right]q_{h,t+1} \\ -I_{r,t+1}\tau_y\left(r_{h,t+1}-\widetilde{\delta}_{h,t+1}\right)\end{array}\right\}\right]
\end{aligned} \tag{8}$$

Labor:

$$\xi_l l_{I,t}^{\vartheta} = \lambda_{I,t}\left(1-\tau_y\right)w_{I,t} \tag{9}$$

Borrowing:

$$1-\mu_t = \beta_I E_t\left[\frac{\lambda_{I,t+1}}{\lambda_{I,t}}\left[1+\left(1-I_{m,t+1}\tau_y\right)r_t-\mu_{t+1}\rho_b\right]\right] \tag{10}$$

Budget constraint:

$$\begin{aligned}
&\left(1+\tau_c\right)c_{I,t}+q_{h,t}\left[h_{I,t}-\left(1-\delta_h\right)h_{I,t-1}\right]+\left(1+r_{t-1}\right)b_{t-1} \\
&\quad\leq w_{I,t}l_{I,t}+b_t-\tau_y\left[w_{I,t}l_{I,t}-\tau_{p,t}q_{h,t}h_{I,t-1}-I_{m,t}r_{t-1}b_{t-1}+I_{r,t}\left(r_{h,t}-\widetilde{\delta}_{h,t}\right)h_{I,t-1}\right] \\
&\qquad-\tau_{p,t}q_{h,t}h_{I,t-1}+tr_{I,t}-\frac{\kappa_h}{2}\left(\frac{h_{I,t}}{h_{I,t-1}}-1\right)^2 q_{h,t}h_{I,t}
\end{aligned} \tag{11}$$

Borrowing constraint:

$$b_t = \rho_b b_{t-1}+\left(1-\rho_b\right)\phi q_{h,t}h_{I,t} \tag{12}$$

1.3 Renter households

Consumption:

$$\frac{1}{c_{R,t}} = \left(1+\tau_c\right)\lambda_{R,t} \tag{13}$$

Housing:

$$r_{h,t} = \frac{\xi_h}{\lambda_{R,t}h_{R,t-1}} \tag{14}$$

Labor:

$$\xi_l l_{R,t}^{\vartheta} = \lambda_{R,t}\left(1-\tau_R\right)w_{R,t} \tag{15}$$

Budget constraint:

$$\left(1+\tau_c\right)c_{R,t}+r_{h,t}h_{R,t-1} = \left(1-\tau_R\right)w_{R,t}l_{R,t}+tr_{R,t} \tag{16}$$

1.4 Non-housing goods producers

Labor:

$$(1-\alpha)\,\psi_P \frac{y_{n,t}}{l_{P,t}} = w_{P,t} \tag{17}$$

$$(1-\alpha)\,\psi_I \frac{y_{n,t}}{l_{I,t}} = w_{I,t} \tag{18}$$

$$(1-\alpha)\,\psi_R \frac{y_{n,t}}{l_{R,t}} = w_{R,t} \tag{19}$$

Capital:

$$\alpha \frac{y_{n,t}}{k_{t-1}} = r_{k,t} + \frac{\kappa_u}{1+\varpi}\left(u_t^{1+\varpi} - 1\right) \tag{20}$$

Utilization rate:

$$\alpha \frac{y_{n,t}}{u_t} = \kappa_u u_t^{\varpi} k_{t-1} \tag{21}$$

Production function:

$$y_{n,t} = z_t \left(u_t k_{t-1}\right)^{\alpha} \left(l_{P,t}^{\psi_P} l_{I,t}^{\psi_I} l_{R,t}^{\psi_R}\right)^{1-\alpha} \tag{22}$$

1.5 Capital and housing producers

Investment in capital:

$$\begin{aligned} q_{k,t} - \kappa_{ik} q_{k,t} \left(\frac{i_{k,t}}{i_{k,t-1}} - 1\right) \frac{i_{k,t}}{i_{k,t-1}} - q_{k,t} \frac{\kappa_{ik}}{2} \left(\frac{i_{k,t}}{i_{k,t-1}} - 1\right)^2 \\ + \beta_P E_t \left[\frac{\lambda_{P,t+1}}{\lambda_{P,t}} \kappa_{ik} q_{k,t+1} \left(\frac{i_{k,t+1}}{i_{k,t}} - 1\right) \left(\frac{i_{k,t+1}}{i_{k,t}}\right)^2\right] = 1 \end{aligned} \tag{23}$$

Law of motion of capital:

$$k_t = (1-\delta_k)\, k_{t-1} + \left[1 - \frac{\kappa_{ik}}{2}\left(\frac{i_{k,t}}{i_{k,t-1}} - 1\right)^2\right] i_{k,t} \tag{24}$$

Investment in housing:

$$\begin{aligned} q_{h,t} - \kappa_{ih} q_{h,t} \left(\frac{i_{h,t}}{i_{h,t-1}} - 1\right) \frac{i_{h,t}}{i_{h,t-1}} - q_{h,t} \frac{\kappa_{ih}}{2} \left(\frac{i_{h,t}}{i_{h,t-1}} - 1\right)^2 \\ + \beta_P E_t \left[\frac{\lambda_{P,t+1}}{\lambda_{P,t}} \kappa_{ih} q_{h,t+1} \left(\frac{i_{h,t+1}}{i_{h,t}} - 1\right) \left(\frac{i_{h,t+1}}{i_{h,t}}\right)^2\right] = 1 \end{aligned} \tag{25}$$

Law of motion of housing:

$$h_t = (1-\delta_h)\, h_{t-1} + \left[1 - \frac{\kappa_{ih}}{2}\left(\frac{i_{h,t}}{i_{h,t-1}} - 1\right)^2\right] i_{h,t} \tag{26}$$

1.6 Fiscal policy

Taxes:

$$\begin{aligned} tax_t = {} & \tau_c c_t + \tau_y \left(w_{P,t} l_{P,t} + w_{I,t} l_{I,t} \right) + \tau_R w_{R,t} l_{R,t} \\ & + \tau_y \left(r_{h,t} - \widetilde{\delta}_{h,t} \right) \left(h_{R,t-1} + I_{r,t} h_{P,t-1} \right) + I_{r,t} \tau_y \left(r_{h,t} - \widetilde{\delta}_{h,t} \right) h_{I,t-1} \\ & + \tau_k \left(r_{k,t} - \delta_k \right) k_{t-1} + \tau_b r_{t-1} \left(b_{t-1} + b^g_{t-1} \right) - I_{m,t} \tau_y r_{t-1} b_{t-1} + \tau_{p,t} \left(1 - \tau_y \right) q_{h,t} h_{t-1} \end{aligned} \tag{27}$$

Transfer payments:

$$tr_{P,t} = \chi_P y_n - \varrho_b b^g_{t-1} \tag{28}$$

$$tr_{I,t} = \chi_I y_n - \varrho_b b^g_{t-1} \tag{29}$$

$$tr_{R,t} = \chi_R y_n - \varrho_b b^g_{t-1} \tag{30}$$

Government debt accumulation:

$$b^g_t = \left(1 + r_{t-1} \right) b^g_{t-1} + g_t + tr_{P,t} + tr_{I,t} + tr_{R,t} - tax_t \tag{31}$$

1.7 Market clearing conditions

Goods market:

$$c_t + i_t + g_t = y_{n,t} \tag{32}$$

Housing market:

$$h_t = h_{P,t} + h_{I,t} + h_{R,t} \tag{33}$$

Total non-housing consumption:

$$c_t = c_{P,t} + c_{I,t} + c_{R,t} \tag{34}$$

Total investment:

$$i_t = i_{k,t} + i_{h,t} \tag{35}$$

Definition of GDP:

$$y_t = \left(1 + \tau_c \right) c_t + p_h h_{t-1} + i_t + g_t \tag{36}$$